PRACTICAL HYDRAULIC FORMULÆ

FOR THE

DISTRIBUTION OF WATER THROUGH LONG PIPES.

E. SHERMAN GOULD.

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DISTRIBUTION OF WATER THROUGH LONG PIPES.

ALSO

NOTES ON WATER SUPPLY ENGINEERING.

BY

E. SHERMAN GOULD,

M. AM. SOC. C. E.

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PREFACE TO SECOND EDITION.

The present edition of this volume consists of two parts. The first is the reproduction of the original work—"Practical Hydraulic Formulæ"—corrected for errors of the press, with a few additional observations and a short chapter on pipe laying. The second contains a concise, but it is hoped comprehensive, outline of the facts and principles involved in the general subject of water supply engineering, together with many of the practical details of hydraulic construction, as gathered by the author during some seventeen years' experience of water works engineering while in the employ of the city of New York and of important private companies.

Some of the leading members of the profession have expressed their approval of "Practical Hydraulic Formulæ" in very flattering terms: It is hoped that in its present extended form the book may merit a still greater degree of favorable recognition than before.

E. S. G.

HAVANA, February, 1894.

TABLE OF CONTENTS.

SECOND PART.

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INTRODUCTION TO FIRST EDITION.

The following pages first appeared as a series of articles in the columns of ENGINEERING NEWS. They are now republished with a few corrections and additions.

In virtue of the law of gravitation, water tends naturally to pass from a higher to a lower level, and without a difference of level there can be no natural flow.

It can be said in all seriousness—although the statement may seem to invite the unjust accusation of an ill-timed attempt at pleasantry—that the whole science of hydraulics is founded upon the three following homely and unassailable axioms :

First. That water always seeks its own lowest level.

Second. That, therefore, it always tends to run down hill, and

Third. That, other things being equal, the steeper the hill, the faster it runs.

In the case of water flowing through long pipes, the hill down which it tends to run is the HYDRAULIC GRADE LINE. If the pipe be of uniform diameter and character, the hydraulic grade line is a straight line joining the water surfaces at its two extremities, provided that the pipe lies wholly below such straight line, and its declivity is measured—like that of all hills —by the ratio of its height to its length.

But if there be any changes whatever in the pipe, either in diameter or in the nature of its inside surface ; or if there be in-

crease or diminution of the volume of water entering it at its upper extremity by reason of branches leading to or from the main pipe, then the hydraulic grade line becomes broken and distorted to a greater or less extent, so that its declivity is not uniform from end to end, but consists of a series of varying grades some steeper than others though all sloping in the same direction.

As regards the third axiom, the proviso—"other things being equal"—must not be overlooked. For we shall find that a pipe of greater diameter but less hydraulic declivity than another, may give a greater velocity to the water passing through it. Also, of two pipes of the same hydraulic slope and diameter, the one having the smoother inside surface affords the greater velocity.

The vertical distance from any point in a pipe to the hydraulic grade line, constitutes the *Piezometric height*, and measures the hydraulic pressure at that point. It will be seen that the solution of problems relating to the flow of water through pipes, lies in the knowing or ascertaining of the piezometric height at any desired point. In general, it is necessary to establish the piezometric height for every point of change of any kind which occurs throughout the entire length of the conduit. The joining of the upper extremities of these heights gives the complete hydraulic grade line.

The object of the following papers is to establish systematic methods for tracing the hydraulic grade line under the different circumstances likely to occur in practice, and generally, to furnish solutions for a large number of practical problems, commencing with the simplest cases and extending to some rather intricate ones, not usually embraced in our hydraulic manuals.

E. S. G.

SCRANTON, Pa., May, 1889.

HYDRAULIC FORMULÆ.

CHAPTER I.

Flow through a Short Horizontal Pipe—Effect on Velocity of Increased Length—Frictional Head—Hydraulic Grade Line—Hydrostatic and Hydraulic Pressures—Piezometric Tubes—Result of Raising a Pipe Line Above the Hydraulic Grade Line—Why the Water Ceases to Rise in the Upper Stories of the Houses of a Town when the Consumption is Increased—Influence of Inside Surface of Pipes Upon Velocity of Flow—Darcy's Coefficients—Fundamental Equations—Length of a Pipe Line Usually Determined by its Horizontal Projection—Numerical Examples of Simple and Compound Systems.

Let us suppose a reservoir of large relative area and depth to be tapped near its bottom by a horizontal cylindrical pipe, of which the length is equal to about three times its diameter.

If there were no physical resistance to the flow, the velocity of the water issuing from the pipe would be given by the formula for the velocity of falling bodies .

$$V = \sqrt{2\,g\,H} = 8.02\ \sqrt{H},$$

in which $V =$ velocity in feet per second, $g =$ the acceleration due to gravity $= 32.2$ ft., and $H =$ the height, expressed in feet, of the surface of the water in the reservoir above the center of the pipe.

Observation shows, however, that in the case cited the velocity of discharge is equal only to that theoretically due to a height of about two-thirds of H; that is :

$$V = \sqrt{\frac{4\,g\,H}{3}} = 6.55\ \sqrt{H}.$$

The remaining third of the height is consumed in overcoming the resistance offered to entry by the edges of the orifice to the inflowing vein of water. The head necessary to overcome the resistance to entry is therefore about one-half of that necessary to produce the velocity of flow.

If the length of the pipe should be increased progressively and indefinitely, the velocity would be found to diminish inversely as the square root of the length. It would correspond, therefore, to a smaller and smaller percentage of the total head H. The resistance to entry diminishes directly as the velocity, and the head necessary to overcome it is always equal to about one-half of that necessary to produce the given velocity as calculated by the laws of falling bodies.

As the length of the pipe (always supposed to remain horizontal) increases, and the velocity of discharge diminishes, the sum of these two heads, *i. e.*, one and a half times that necessary to produce the actual velocity, is no longer equal to the total head H, as we have seen to be the case when the length of the pipe is only about three diameters. What, then, becomes of the remainder of H? It is consumed in overcoming the increasing frictional resistances engendered by contact of the moving water with the inside surface of the pipe. When the pipe is very long, and the velocity therefore relatively low, the sum of the velocity and entrance heads is small, and by far the greater part of the total head is required to force the water through the pipe against the opposition offered by friction to its flow. In such cases, which are those occurring most generally in practice when water is conveyed from a reservoir for the supply of a town, the velocity and entrance heads are commonly ignored, and the total head H is supposed to be available for overcoming the frictional resistances. As this occasions, however, an error—although generally a very small one—in the *wrong direction*, judgment is required in exercising this latitude. Later on we will revert to this point,

but for the present we will consider only frictional resistances, particularly since—and indeed because—in practice our assumed data are almost always sufficient to afford an ample margin to cover the neglected factors.

In what precedes we have considered a horizontal pipe issuing from a reservoir in which the surface of the water is maintained at a constant level. In practice these conditions rarely obtain.

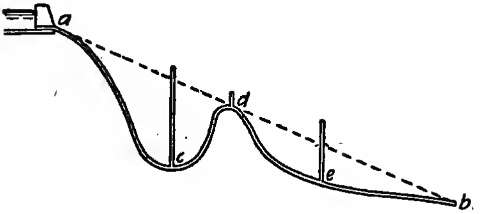

FIG. 1.

Suppose a system, such as is shown by Fig. 1, consisting of a reservoir and pipe line of varying and contrary slopes. As the level of the water in the reservoir would be subject to fluctuations, and liable at times to be greatly drawn down, it is customary to consider the surface of the water as standing at its lowest possible level, *i. e.*, the mouth of the pipe. In this case, the value of H would be equal to the difference of level of the two extremities a and b of the pipe, and the line ab joining the centers of the two ends would form what is called *the hydraulic grade line*, the establishing of which is the first step to be taken in laying out a system of water supply.

Suppose that at the points c, d, and e vertical tubes, open at their upper ends, were connected with the pipe. The water, when flowing freely from the end b of the pipe would rise in each of these tubes to about the height of the hydraulic grade line at these points, and, if branches were connected at the points c, d, and e, they would, when closed, sustain a pressure upon their

gates equal to the head comprised between the gates and the grade line. If the gates were open, the branches would discharge water under heads equal to the difference of level of the hydraulic grade line at the point of embranchment and their remote extremities, less a certain amount depending upon the volume discharged, which will be spoken of hereafter.

At *d*, where the top of the pipe just touches the grade line, there would be no pressure at all when the water was flowing through the pipe, except the very small amount due to the depth of water in the pipe itself.

If the end *b* should be closed so that there was no movement of water in the pipe, the water would rise in the tubes, if they were long enough, until it stood at the same level as the water in the reservoir, and the pressures at *c*, *d*, and *e* would be equal to the head comprised between these points and the level of the water in the reservoir. The latter is called the *hydrostatic pressure*, or simply the *static pressure*, and the former the *hydraulic pressure*, at these points.

The tubes spoken of are known by the name of *piezometric tubes*.

The importance of correctly establishing the hydraulic grade line is illustrated by reference to a case such as is shown in Fig. 2, in which the pipe, at the point *c*, rises above the grade line *ab*. To explain: It will be readily deduced from what has been already said in reference to horizontal pipes that the velocity of flow, and consequently the delivery, of a pipe increases with the steepness of its slope. In this case the pipe *ab* is divided into two parts, the one *ac* with a hydraulic grade line flatter than *ab*, and the other *cb* with one steeper than *ab*. The delivery of the entire system, if the pipe were of the same diameter throughout, would be governed by the flatter portion *ac*, and the portion *cb* would be capable, in virtue of its steeper slope, of discharging a greater

volume of water than it could receive from *ac*. Consequently it would act merely as a trough and would never run full, and if a piezometric tube were placed in it at *d* for instance, no water would rise in the tube, and no pressure be exerted.

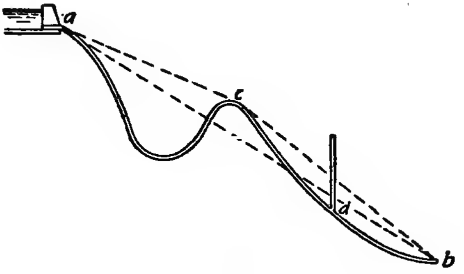

FIG. 2.

It is very important, therefore, in locating a pipe line that the pipe should nowhere rise above the hydraulic grade line. The full amount of water could indeed be carried over the high point *c* by means of siphonage, but this expedient is not resorted to in practice. Should the nature of the ground require such a location as that shown in Fig. 2, it would be necessary to increase the diameter of the pipe between *a* and *c*, so that it would deliver the required volume under the reduced head, and to diminish that between *c* and *b*, so that it should only deliver the same volume under its increased head, and therefore run full. The calculations necessary to determine the proper diameters will be shortly developed.

Should the axis of the pipe coincide exactly with the hydraulic grade line *ab*, the pipe would run full (provided the feed were sufficient) but would be under no pressure, and no water would rise in piezometric tubes placed on any part of the pipe. Moreover, as the slope would be the same for any portion of the pipe, the velocity and delivery would be unchanged, whether we

cut the pipe off at a comparatively short length, or extend it indefinitely.

As a further and very interestiug practical illustration of the effects of a hydraulic grade line of varying steepness, let us con-

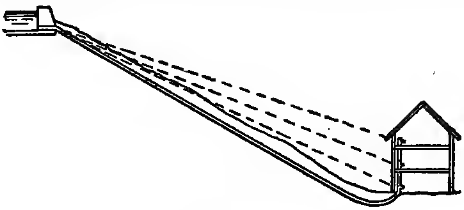

FIG. 3.

sider (Fig. 3) the case of a house supplied with water by a pipe communicating with a reservoir.

Suppose the pipe to be sufficiently large to furnish a certain volume of water per hour to the upper story of the house. If now a larger volume were required, it is clear that, unless we increase the diameter of the pipe, it would be necessary to increase the steepness of pitch of the grade line, in other words, to increase the head, or difference of level between the reservoir and the point of discharge. The increased volume could therefore be only drawn from a lower story.*

Or, to put the same conditions under a different form, suppose, as before, the pipe to be just large enough to supply the top story of the house, the taps on the lower floors being closed. Should they be opened, it is evident that a greater amount of water would be discharged from them than from the upper one, because they would discharge under a greater head. The result would be a diminished flow or perhaps no flow at all on the top floor, and an increased discharge of water at a lower level.

* In other words, if we wish to increase the *volume*, the diameter of pipe remaining constant, we must increase the *velocity;* and the increased velocity can only be obtained by an increased difference of level between the two ends of the pipe. If the elevation of the upper end, or surface of water in the reservoir, cannot be increased, that of the lower end, or point of discharge, must be diminished.

This case shows why the water ceases to rise in the upper stories of the houses of a town when the consumption increases.

It has been found by observation that the velocity of water flowing through pipes is greatly affected by the nature of their inside surface, increasing with the smoothness and diminishing with the roughness of the same. By direct experiment, coefficients have been established for different conditions of surface. It has also been found that these coefficients vary slightly with the diameter of the pipe, a pipe of a certain size giving a greater velocity than one of the same character of inside surface but of smaller diameter, the differences becoming smaller as the diameters increase.

The value of this coefficient, which will be designated throughout this paper by *C*, is given below for a number of different diameters and for two classes of pipes,—those which are clean and smooth on the inside, and those which are rough and incrusted, the difference being as 2 to 1. As all pipes, after a few years of service, are liable to become more or less roughened and obstructed by deposits, it is always safer when calculating the proper diameters of a permanent water supply, to assume rough pipes at once, although diameters thus calculated will, for perhaps a number of years, deliver quantities greatly in excess of the desired amounts.

The coefficients given below are those determined experimentally by DARCY. Of course, in the snbsequent calculations which will be made, any other values might be substituted for the ones given. It is well to remark, however, in regard to the coefficient, that although this factor is a controlling one in the calculation of the discharge of pipes, it is nseless to attempt an excessive refinement in establishing its value, because not only is it difficult to determine this value with exactness for a given diameter and condition of pipe, but this condition, and even the

diameter of the pipe, is liable to undergo considerable variation in the same pipe in the course of a few years.

TABLE OF COEFFICIENTS.

Diameter in inches.	*Value of C for rough pipes.*	*Value of C for smooth pipes.*
3	0.00080	0.00040
4	0.00076	0.00038
6	0.00072	0.00036
8	0.00068	0.00034
10	0.00066	0.00033
12	0.00066	0.00033
14	0.00065	0.000325
16	0.00064	0.00032
24	0.00064	0.00032
30	0.00063	0.000315
36	0.00062	0.00031
48	0.00062	0.00031

In all the following calculations, the coefficient for rough pipes will be used.

The two fundamental equations relating to the flow of water through long pipes are :

$$\frac{D \times H}{L} = C V^2 \qquad (1)$$

$$Q = A V \qquad (2)$$

Equation No. 2 will generally be written :

$$Q = A \sqrt{\frac{D \times H}{C \times L}} \qquad (3)$$

by taking the value of V from (1).

The first of these has been established by DARCY ; the second is based upon a self-evident proposition.

In these equations :

D = diameter of pipe in feet
H = total head " "
L = length of pipe " "
C = coefficient
V = mean velocity in feet per second
Q = discharge in cubic feet per second
A = area of pipe in square feet = $D^2 \times 0.785$

The above two formulæ solve, directly or indirectly, all prob-

lems relating to the flow through long pipes, and all such problems must be brought into a form admitting of their application, in order to obtain a solution.

It will be observed that $\frac{H}{L}$ is the rise or fall per foot of length of pipe, and is therefore the natural sine of the inclination of the slope to the horizon. This relation is frequently used under the form $I = \frac{H}{L}$. Using this notation, (1) would be written:

$$D I = C V^2$$

In long pipes the length is generally taken as being equal to the horizontal distance separating the two ends of the pipe, as the difference between this distance and the actual length of the pipe is relatively insignificant. If, however, a case should present itself in which this difference was considerable, the actual length of pipe should be taken. Further on, an extreme case of this kind will be given, presenting some interesting features.

Some practical examples of the use of these formulæ will now be given. In all that follows, the resistances of entry, exit, and velocity will be neglected, and the total head will be considered as available for overcoming friction. The examination of cases where the above factors are included is reserved for a later portion of this paper, as they are of secondary importance when dealing with long pipes.

Example 1.—A pipe, 1 ft. in diameter and 1,000 ft. long, has a total fall of 10 ft. What are the velocity and volume of its discharge ?

Substituting the given values in (1) we have :

$$\frac{1 \times 10}{1000} = 0.00066\ V^2$$

$$V = 3.89 \text{ ft. per second.}$$

Using this value of V in (2), we have:

$$Q = 0.785 \times 3.89$$
$$Q = 3.055 \text{ cu ft. per second.}$$

Example 2.—Two reservoirs, having a difference of level of water surface of 30 ft., are joined by a pipe 3,000 ft. long. What should be the diameter of the pipe to deliver 16 cu. ft. of water per second from the upper to the lower reservoir?

Eliminating V between (1) and (2) we have:

$$\frac{D \times H}{L \times C} = \frac{Q^2}{A^2}.$$

Observing that $A = D^2\ 0.785$;

$$\frac{D \times H}{L \times C} = \frac{Q^2}{D^4 \times 0.616}$$

Whence

$$D^5 = \frac{Q^2 \times L \times C}{H \times 0.616} \tag{4}$$

If we knew the proper value of the coefficient C in the above equation, it could be immediately solved, and the value of D obtained. But C varies with the diameter, and the diameter is as yet unknown. We must therefore have recourse to "Trial and Error" for a solution.

Suppose it should appear to us, at first sight, that a 12-in. pipe was likely to be of the proper size. We therefore take $C = 0.00066$, and write:

$$D^5 = \frac{256 \times 3000 \times 0.00066}{30 \times 0.616}$$
$$D^5 = 27.70$$
$$D = 1.94 \text{ ft.}$$

From this we see that the pipe should be nearly 2 ft. in diameter, and as we have taken too large a coefficient (that for 24 ins. = 0.00064), we are sure that 1.94 is too large. As pipes are never made of fractional diameters, the above value of D would be taken = 24 ins., and therefore we would push the calculation no further. If the case had happened to be one requiring minute accuracy, we would recalculate the above equation, using 0.00064 for the value

of C. The result would be, $D = 1.93$ ft. nearly, practically the same as the value already obtained.

The above examples (which are those commonly occurring in practice) are very simple, and involve only the direct application of the fundamental formulæ. Let us now consider cases of a more complicated character, where they can only be used indirectly, and where a certain amount of judgment and tact is required in the preparation of the data.

Example 3.—Suppose a reservoir R (Fig. 4) containing a depth of water of 50 ft. above the center of the horizontal pipe A, 1 ft. in diameter and 1,000 ft. long, connected by a reducer with another horizontal pipe B, 2 ft. in diameter and 3,000 ft. long. It is required to calculate the piezometric head h at the junction, from which the discharge can be calculated, and the hydraulic grade line abc established.

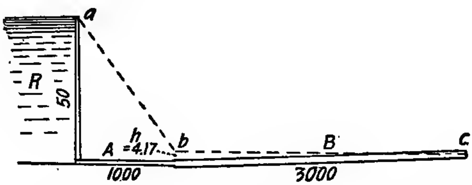

FIG. 4.

It is evident that the 24-in. pipe must, under the head h, discharge the same quantity per second as the 12-in. pipe, under the head $50 - h$. We have then from (3) the equality:

$$3.14 \sqrt{\frac{2 \times h}{3000 \times 0.00064}} = 0.785 \sqrt{\frac{1 \times (50 - h)}{1000 \times 0.00066}}.$$

Dividing by 0.785, squaring, and simplifying:

$$\frac{h}{0.02} = \frac{50 - h}{0.22},$$

whence

$$h = 4.17.$$

We can now very readily get the discharge, by substituting

the valne 4.17 for h in either member of the above equality. Thus :

$$Q = 3.14 \sqrt{\frac{4.17}{0.96}} = 6.54 \text{ cu. ft. per sec.}$$

Verifying in the other member—a precaution which should never be neglected—we obtain the same result.

It is evident that the diameter of B may be assumed so large that no value of h can be found to satisfy the condition that both pipes shall run full with the given height of water in the reservoir. In such a case the pipe B serves only as a trough to receive the water discharged through A under a head of 50 ft.

Suppose that in the above example the places of the two pipes, A and B, should be changed. Evidently we should have:

$$h = 45.83.$$

This piezometric height would give, with the transposed position of the pipes, the same discharge as before, the only difference being a notable change in the hydraulic grade line. If the pipes were tapped by branches, the greater elevation of the grade line in this case would bring a much greater pressure upon the branches, enabling them to deliver water at a higher level than in the first position of the pipes.

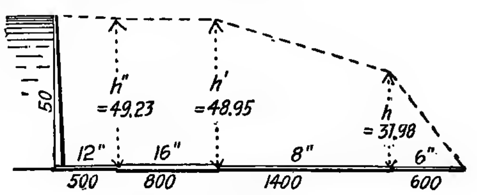

FIG. 5.

The above example may be extended so as to cover cases where pipes of several different diameters are used. Thus, suppose a system of pipes, such as is shown in Fig. 5, where a reservoir with a head of 50 ft. of water, as before, is tapped by a horizontal line of pipes, consisting in order of 500 ft. of 12-in., 800 ft. of 16-in., 1,400 ft. of 8-in., and 600 ft. of 6-in. pipe.

This example may be worked in the same way as the previous one, by getting equations for h, h', and h'' expressed, by substitution, in terms of h. But it will be easier to treat the question in another way, which will also exhibit the further resources which we have at our disposal in solving hydraulic problems.

Since each section of pipe must discharge equal volumes in equal times, it is evident that the respective velocities of flow must vary inversely as the areas of the pipes. These areas vary as the squares of the different diameters. Designating, therefore, by V the lowest rate of velocity, *i. e.*, that of the water passing through the largest pipe (the 16-in. one), we obtain the relative velocities in the other pipes by multiplying V by the ratio of the square of the diameter of the largest pipe to the squares of the other diameters. It will be convenient to form the following table:

Lengths in ft.	Diameters in ft.	Velocities in ft. per second.
500	1	1.78 V
800	$1\frac{1}{3}$	V
1,400	$\frac{2}{3}$	4 V
600	$\frac{1}{2}$	7.11 V

Beginning at the lower end of the system, that is with the 6-in. pipe, and employing formula (1) in which h and V are the unknown quantities, we have:

$$\frac{1}{2} \times \frac{h}{600} = 0.00072 \times (7.11)^2 \times V^2;$$

whence:
$$h = 43.68\ V^2$$

again:
$$\frac{2}{3} \times \frac{(h' - h)}{1400} = \frac{2}{3} \times \left(\frac{h' - 43.68\ V^2}{1400}\right) = 0.00069 \times (4)^2 \times V^2;$$

whence:
$$h' = 66\ 86\ V^2$$

similarly:
$$\frac{4}{3} \times \frac{(h'' - h')}{800} = \frac{4}{3} \times \left(\frac{h'' - 66.86\ V^2}{800}\right) = 0.00065 \times V^2$$

whence:
$$h'' = 67.25\ V^2$$

Finally:
$$\frac{50 - h''}{500} = \frac{50 - 67.25\ V^2}{500} = 0.00066 \times (1.78)^2 \times V^2$$

whence:
$$V^2 = 0.7321$$
$$V = 0.8556 \text{ ft. per second.}$$

Substituting this value of V^2 in the above equations :

$$h = 31.98 \text{ ft.}$$
$$h' = 48.95 \text{ "}$$
$$h'' = 49.23 \text{ "}$$

We also get the velocities in the different pipes, thus :

6 inch,	velocity	$= 7.11 \times 0.856 = 6.086$
8 "	"	$= 4 \times 0.856 = 3.424$
16 "	"	$= 1 \times 0.856 = 0.856$
12 "	"	$= 1.78 \times 0.856 = 1.524$

The work can be checked by using the above values of h, h' and h'', along with the other data, in (1), and obtaining the velocities in this way.

Thus, beginning with the 6-in. pipe :

$$\frac{1}{2} \times \frac{31.98}{600} = 0.00072\, V^2$$
$$V = 6.08$$
$$\frac{2}{3} \times \frac{16.97}{1400} = 0.00069\, V'^2$$
$$V' = 3.42$$
$$\frac{4}{3} \times \frac{0.28}{800} = 0.00065\, V''^2$$
$$V'' = 0.85$$
$$1 \times \frac{0.77}{500} = 0.00065\, V'''^2$$
$$V''' = 1.53$$

A very close agreement throughout.

In the above calculations the decimals have been carried out further than would ordinarily be necessary in practice. It was done in the present instance in order to avoid discrepancies in checking.

We have another check, in the volumes discharged. Thus the discharge through the 6-in. pipe, with the given velocity, is by (2):

$$Q = 0.195 \times 6.086$$
$$Q = 1.19 \text{ cubic ft. per second.}$$

All the other pipes should have an equal discharge; for instance, the 12-in. pipe gives :

$$Q = 0.78 \times 1.524$$
$$Q = 1.19 \text{ cubic ft. per second.}$$

CHAPTER II.

Calculations are the Same for Pipes laid Horizontally or on a Slope—Qualification of this Statement—Pipe of Uniform Diameter Equivalent to Compound System—General Formula—Numerical Example—Use of Logarithms (foot note)—Numerical example of branch pipe—Simplified method—Numerical Examples—Relative discharges through branches variously placed—Discharges determined by plotting—Caution regarding results obtained by calculation—Numerical examples.

In the preceding examples a series of horizontal pipes has been considered, the head being produced by an elevated reservoir placed at one end. The results would have been identical, however, if the head had been produced by the pipes being laid upon a slope, provided the difference of level between the two extremities remained the same, for the velocities and hydraulic grade line would remain unaltered. The pressure in the pipes would vary however, according to their distance below the hydraulic grade line, the pressure being measured at any given point in the pipe line, by the vertical distance between such point and the grade line. If the pipes were laid exactly upon the hydraulic grade line there would be no pressure at all in the pipes, and if they rose at any point above it, there would be either no flow or a diminished one, unless siphonage were resorted to.

In order to make this point very plain, we will consider the same system of pipes as that used in the last example, but laid as shown in Fig. 6. the upper extremity being fed by a constant supply, with only head enough to overcome resistance to entry, and produce initial velocity, which will be treated of further on.

Calculating precisely as before, we get the same hydraulic

grade line, unbroken by the rising grade of the last 200 ft. of 6-in. pipe.

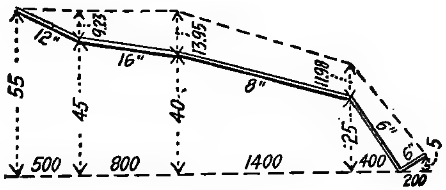

FIG 6.

It is sometimes desirable to ascertain the uniform diameter of a pipe which shall be equivalent to a series of pipes of different diameters, such as we have just been studying. This may be done by an application of formula (4), which for this purpose is written in the following form:

$$H = \frac{C\,Q^2}{0.616} \times \frac{L}{D^5}$$

As an example, let us calculate the diameter of a single pipe, of the same total length and fall as the series of pipes which we have just had under consideration, and capable of discharging an equal volume. We will first establish the general formula for all such problems, expressing the difference of piezometric level between the two ends of each pipe respectively, by h_1,h_2,h_3,h_4, etc., their respective lengths by l_1,l_2,l_3,l_4, etc., their respective diameters by $d_1 d_2 d_3 d_4$, etc., and their respective coefficients by $c_1 c_2 c_3 c_4$, etc., commencing with the lower end. We will express the total length by L, the total difference of level by H, the unknown diameter by D, and its coefficient by C.

Now, observing that the quantity discharged per second by each pipe is the same, we have the 4 equations:

$$h_1 = \frac{c_1\ Q^2}{0.616} \times \frac{l_1}{d_1^5}$$

$$h_2 = \frac{c_2\ Q^2}{0.616} \times \frac{l_2}{d_2^5}$$

$$h_3 = \frac{c_3 \ Q^2}{0.616} \times \frac{l_3}{d_3^5}$$

$$h_4 = \frac{c_4 \ Q^2}{0.616} \times \frac{l_4}{d_4^5}$$

Adding, and observing that the sum of the partial heads $h_1 \ h_2 \ h_3 \ h_4$ equals H, we have :

$$H = \frac{Q^2}{0.616} \left(\frac{c_1 \, l_1}{d_1^{\,5}} + \frac{c_2 \, l_2}{d_2^{\,5}} + \frac{c_3 \, l_3}{d_3^{\,5}} + \frac{c_4 \, l_4}{d_4^{\,5}} \right)$$

but we have also the equation

$$H = \frac{C \, Q^2}{0.616} \times \frac{L}{D^5}$$

whence, suppressing the common factor :

$$\frac{C \, L}{D^5} = \frac{c_1 \, l_1}{d_1^{\,5}} + \frac{c_2 \, l_2}{d_2^{\,5}} + \frac{c_3 \, l_3}{d_3^{\,5}} + \frac{c_4 \, l_4}{d_4^{\,5}} \qquad (5$$

The above is the general formula.

Substituting the special values of our example :

$$\frac{3300}{D^5} \times C = \frac{0.33}{1^5} + \frac{0.52}{(\frac{4}{3})^5} + \frac{0.966}{(\frac{2}{3})^5} + \frac{0.432}{(\frac{1}{2})^5}$$

Giving a preliminary approximate value to C of 0.00066, we have

$$\frac{2.178}{D^5} = 0.33 + 0.123 + 7.335 + 13.824$$

$$D^5 = 0.1007$$

$$D = 0.63$$

This value of D indicates a practical diameter of 8 ins.

In order to check this value, we may write (4) under the form :

$$Q = \sqrt{\frac{D^5 \times H \times 0.616}{L \times C}}$$

Substituting given values :

$$Q = \sqrt{\frac{0.1007 \times 50 \times 0.616}{2.178}}$$

$$Q = 1.193 \text{ cu. ft. per second,}$$

thus proving the correctness of the work.

These calculations can be abridged, and, in many cases, sufficient accuracy secured by adopting a mean common value for C. If we do so in the present case, C becomes a common factor, and disappears from the calculation, (5) becoming

$$\frac{L}{D_5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}, \text{ etc.} \qquad (5) \textit{ bis}$$

If this equation be worked out for the above given values, we have :

$$D = 0.64$$

or 8 ins. as before.

It will be observed that this process might have been used with advantage in the previous example, by ascertaining the discharge of an equivalent pipe, and then calculating the heads necessary to produce this discharge through the different pipes.

In calculating fifth powers and roots, a table of logarithms is almost indispensable. If none is at hand a table of squares and cubes is of some use, remembering that a number can be raised to the fifth power by multiplying together its square and cube. Fifth roots, in the absence of logarithms, can only be extracted by "trial and error," using the above rule for fifth powers.*

Example 4th. A horizontal pipe (Fig. 7), 48 ins. in diameter and 2,000 ft. long, issues from a reservoir in which the surface of the water is maintained at a constant height of 50 ft. above the center of the pipe. Midway, this pipe is tapped by a branch pipe 24 ins. in diameter and 500 ft. long, with a rising grade of 4 ft. in 500. What is the piezometric head h at the junction, and what the discharge from each pipe ?†

It is evident that the 48-in. pipe above the junction must, with the head 50—h, discharge as much water per second as the

* All hydraulic calculations are greatly facilitated by the use of logarithms; and those engaged in making such calculations should not fail to familiarize themselves with the use of these powerful auxiliaries to arithmetical work.

† With these lengths and diameters, the above system does not properly come under the classification of "long pipes." As the present object is only to exemplify methods of calculation, the example is equally good.

combined discharge of the 48-in. pipe below the branch with the head h, and the 24-in. pipe with the head h—4. From (3),

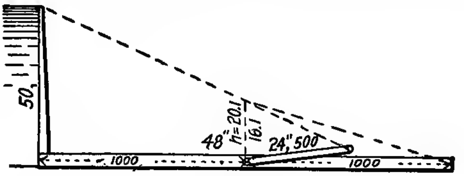

FIG. 7.

which in this case will perhaps be the most convenient equation for quantity, though that derived from (4) is frequently useful, we have :

$$Q = 12.56 \sqrt{\frac{4\,(50 - h)}{1000 \times 0.00062}}$$

$$q = 12.56 \sqrt{\frac{4\,h}{1000 \times 0.00062}}$$

$$q' = 3.14 \sqrt{\frac{2\,(h - 4)}{500 \times 0.00064}}$$

which, put in equation, give :

$$12.56 \sqrt{\frac{4\,(50 - h)}{1000 \times 0.00062}} =$$

$$12.56 \sqrt{\frac{4\,h}{1000 \times 0.00062}} + 3.14 \sqrt{\frac{2\,(h - 4)}{500 \times 0.00064}}$$

The coefficients 0.00062 and 0.00064 are so nearly equal that we may, in the following calculations, discard them as common factors. Dividing by 3.14 and striking out also the common factors $\frac{4}{1000}$ and $\frac{2}{500}$, we have simply :

$$4\,\sqrt{50 - h} = 4\,\sqrt{h} + \sqrt{h - 4}$$

Squaring $$800 - 16\,h = 16\,h + h - 4 + 8\,\sqrt{h^2 - 4\,h}$$

which gives : $$33\,h = 804 - 8\,\sqrt{h^2 - 4\,h}$$

Neglecting, for a first approximate value of h the quantities affected by the radical :

$$33h = 804$$

Neglecting decimals:

$$h = 24.$$

Substituting this value for h under the radical:

$$33h = 804 - 8\sqrt{576 - 96}$$

which gives, always neglecting decimals, a second approximate value:

$$h = 19.$$

A third and fourth approximation give respectively $h = 20.3$ and $h = 20$.

We will take 20.1 as very near the true value.*

Substituting 20.1 in place of h in the equations giving the quantities discharged, we have:

$$Q = 12.56\sqrt{\frac{4 \times 29.9}{0.62}} = 174.45$$

$$q = 12.56\sqrt{\frac{4 \times 20.1}{0.62}} = 143.05$$

$$q' = 3.14\sqrt{\frac{2 \times 16.1}{0.32}} = 31.50$$

We have thus:

$$Q = q + q'.$$

The above method gives directly the true value of h; but it involves tedious figuring, even in our example, which happens to admit of many simplifications owing to the number of common factors. It will be easier, and often shorter, to obtain the value of h by first *assuming* one which we judge likely to be near the truth, calculating what discharge it would give from the two branches, and then calculating the head necessary to discharge the same quantity from the single pipe above the branch. Then, comparing the total height thus obtained with the known height of the water in the reservoir, we can deduce the true value of h by a proportion.

Let us apply this method to the above example. We know

* The value of h may be obtained directly by using the usual formula for adfected quadratics; but with the aid of a table of squares and square roots, the above approximate method will generally be the easier and quicker one.

at once that h must be less than 25, because that would be its value if the 24-in. branch were closed. Supposing we judged that 22 ft. would be about correct. We then have to solve the two equations:

$$q = 12.56 \sqrt{\frac{4 \times 22}{0.62}} = 149.60$$

$$q' = 3.14 \sqrt{\frac{2 \times 18}{0.32}} = 33.30$$

also, for the equal discharge through the 48-in. pipe above the branch, squaring (3), we have:

$$h = \frac{(182.90)^2 \times 0.62}{(12.56)^2 \times 4} = 32.87$$

This height, added to 22, the assumed value of h, gives a total height of 54.87 ft. as against 50 ft., the actual total height. By proportion we have:

$$\frac{h}{22} = \frac{50}{54.87}$$

This value of h agrees with that already found.

If the 24-in. branch were closed we should have for the discharge:

$$Q = 12.56 \sqrt{\frac{4 \times 50}{1.24}} = 159.51$$

When the 24-in. branch was open we had a total discharge of 174.73 cu. ft. per second. There is an increase, therefore, of about 9½ per cent. by opening the branch.

Let us now see what the discharge would be if the branch were placed only 500 ft. from the reservoir, instead of 1,000 ft., all the other conditions remaining the same.

We will assume $h = 33$ ft. and solve the two equations

$$q = 12.56 \sqrt{\frac{4 \times 33}{1500 \times 0.00062}} = 149.5$$

$$q' = 3.14 \sqrt{\frac{2 \times 29}{0.32}} = 42.3$$

also
$$h' = \frac{(191.8)^2 \times 0.31}{(12.56)^2 \times 4} = 18.07$$

giving a total height of 51.07 as against 50. Reducing :

$$\frac{h}{33} = \frac{50}{51.07}$$
$$h = 32.3$$

Using this value, instead of the assumed one, we have :

$$12.56\sqrt{\frac{4 \times 17.7}{0.31}} = 12.56\sqrt{\frac{4 \times 32.3}{0.93}} + 3.14\sqrt{\frac{2 \times 23.3}{0.32}}$$
$$189.83 = 148.03 + 41.76$$

very nearly.

As compared with the discharge when the 24-in. branch is closed this shows a gain of 19 per cent., just double the gain when the branch was located at the center of the pipe.

Supposing now that the branch were placed 1,500 ft. from the reservoir. Assuming 10 ft. as a probable value of h we have :

$$q = 12.56\sqrt{\frac{4 \times 10}{500 \times 0.00062}} = 142.46$$

$$q' = 3.14\sqrt{\frac{2 \times 6}{0.32}} = 19.23$$

also:
$$h' = \frac{(161.7)^2 \times 0.93}{(12.56)^2 \times 4} = 38.53$$

By proportion
$$\frac{h}{10} = \frac{50}{48.53}$$
$$h = 10.30$$

Using this value instead of the assumed one :

$$12.56\sqrt{\frac{4 \times 39.7}{0.93}} = 12.56\sqrt{\frac{4 \times 10.3}{0.32}} + 3.14\sqrt{\frac{2 \times 6.3}{0.32}}$$
$$164.13 = 144.57 + 19.68$$

very nearly.

As compared with the discharge when the 24-in. branch is closed, this shows a gain of not quite 3 per cent., which is in marked contrast to the gain when the branch was only 500 ft.

from the reservoir, being less than one-sixth of the gain, in that case.

It will be interesting to study a little more in detail the question of relative discharges. We have seen that when there is no branch open on the 48-in. pipe, its discharge is 159.51 cu. ft. per second. The 24-in. branches, wherever placed, increase the total discharge, but diminish that in the 48-in. pipe, below the branch. By comparing the above quantities, it will be perceived that the flow from the 48-in. pipe is diminished approximately by that proportion of the quantity flowing through the 24-in. branch which is represented by its proportionate distance from the reservoir. Thus, when the branch is 1,500 ft., or three-quarters of the length of the 48-in. pipe, from the reservoir, as in the last case, its discharge is 19.62 cu. ft. per second. Three-quarters of this quantity is 14.715, which, subtracted from 159.51, leaves 144.795, or very nearly that of the 48-in. pipe below the branch, as determined by calculation.

In the same way half of the discharge, when the branch is situated half way from the reservoir, subtracted from 159.51, gives also very nearly the amount discharged below the branch. When the branch is 500 ft., or one-quarter of the total distance, from the reservoir, one-quarter of its discharge taken from 159.51 gives very closely the discharge as calculated for the 48-in. pipe below the branch.

Let us now take an extreme position for the branch, and suppose it placed close to the reservoir, so that there is practically no portion of the 48-in. pipe between it and the reservoir. There will, therefore, be no part of the flow from the branch subtracted from that of the main pipe, and the two will each discharge the same quantity as if the other were not there. That is, the 48-in. pipe will discharge 159.51, and the 24-in. 53.24 cu. ft. per second.

If we should take another extreme position for the branch, and suppose it placed at the end of the 48-in. pipe, it is obvious that, with its assumed rising grade of 4 ft. in 500, it would dis-

charge no water at all. A position could be found by trial where it would just cease to discharge water, but for the object of the present investigation this is not necessary.

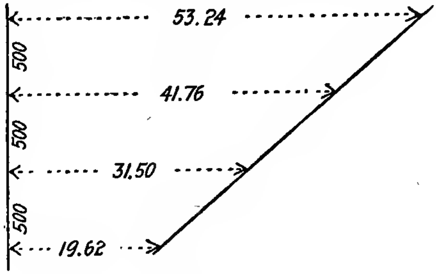

FIG. 8.

If the above results are plotted, as in Fig. 8, a very instructive diagram is obtained. The successive 500 ft. lengths being laid off as abscissæ, and the discharges measured upon the corresponding ordinates, it will be seen that their extremities all lie nearly in the same straight line. If, therefore, the discharges for any two positions of the branch be calculated, and the straight line drawn passing through their extremities, the discharge for any other position of the branch can be obtained by erecting an ordinate at the given point to the straight line, and the flow through the main also obtained by subtracting the proper portion of that of the branch.

In practice, when making calculations similar to those under consideration, one error must be carefully guarded against, namely, the supposing that the actual results will be exactly as calculated. The chief value of these calculations lies in the fact that they furnish pretty trustworthy relative results, that is, they establish fairly well in practice the fact that if a certain pipe delivers a certain volume of water in a certain position, it will deliver a certain greater or less amount in another. The actual amounts, in either case, cannot be surely determined, as they de-

pend upon so many varying circumstances about which, even when aware of their existence, we have no exact data.

Let us next suppose a system in which the 48-in. pipe is tapped every 500 ft. by a 24-in. pipe, 500 ft. long, laid as before with a grade of 4 ft. in 500.

Assuming a height of 9 ft. for the piezometric column h nearest the free end of the pipe we have :

$$12.56\sqrt{\frac{4\times 9}{0.31}}+3.14\sqrt{\frac{2\times 5}{0.32}}=12.56\sqrt{\frac{4\,(h'-9)}{0.31}}$$

Since the denominators under the radicals are so nearly equal we may cancel them, and making other simplifications, write :

$$\sqrt{9}+\frac{1}{8}\sqrt{10}=\sqrt{h^1-9}$$

Whence: $h'=20.52$

Again: $\sqrt{11.52}+\frac{1}{8}\sqrt{33.04}=\sqrt{h''-20.52}$

$$h''=37.43$$

Also $\sqrt{16.91}+\tfrac{1}{8}\sqrt{66.86}=\sqrt{h'''-37.43}$

$$h'''=63.79$$

By proportion we have: $\dfrac{h}{9}=\dfrac{50}{\cancel{63.97}\ 63.79}$

$$h=7.05$$

As the value of $h'''=63.79$ differs considerably from the true value $=50$, and as the above proportion is not exactly absolute, particularly in a somewhat complex system like the present, it is probable that the value just obtained for h is not a sufficiently close approximation to answer our purpose. We will therefore make a second calculation, using 7 as a second approximate value for h.

Carrying the calculation through precisely as above, we obtain the following values :

$$h=7.32$$
$$h'=16.42$$
$$h''=29.60$$
$$h'''=50.00$$

Calculating the various discharges under these piezometric heads, calling those through the different sections of 48-in. pipe, commencing at the lower end, Q, Q', Q'', Q''', and those through the corresponding 24-in. branches, q, q', q'', we have :

$$
\begin{aligned}
Q &= 122.05 \\
q &= 14.30 \\
\hline
Q + q &= 136.35 \\
Q' &= 136.10 \\
q' &= 27.62 \\
\hline
Q' + q' &= 163.72 \\
Q'' &= 163.75 \\
q'' &= 39.72 \\
\hline
Q'' + q'' &= 203.47 \\
Q''' &= 203.75
\end{aligned}
$$

These results show a very close agreement.

It is worthy of note that the total discharge in this case is not greatly increased over that obtained with a single branch situated 500 feet from the reservoir. In general it will be found, as in these two cases, that when a main is tapped at a certain point by a single branch, the total discharge is comparatively but slightly increased by the introduction of a series of similar branches placed below the first junction. The position of the first branch, however, has, as the above examples show, a very great influence both on the volume of discharge and the form of the hydraulic grade line. This latter feature merits careful attention.

It will be interesting to study the effect upon the flow through such a system as we have been just considering, when the conditions are somewhat changed. For instance, in the last example let us suppose that the three branch pipes, instead of having each an equal rising grade of 4 feet in their length of 500 feet, have rising grades respectively of 4 feet, 12 feet and 24 feet in 500, commencing at the lower branch, all other conditions remaining the same.

Assuming, as before, an approximate value for h of 9 feet, we get, as before

$$h' = 20.52$$

Our next equation will be :

$$\sqrt{11.5} + \frac{1}{8}\sqrt{17.04} = \sqrt{h'' - 20.52}$$

$$h'' = 35.81$$

Again :

$$\sqrt{15.29} + \frac{1}{8}\sqrt{23.62} = \sqrt{h''' - 35.81}$$

$$h''' = 56.24$$

This value is sufficiently near the given one of 50, to warrant our using it to obtain pretty close approximate values, by proportion, as follows :

$$\begin{aligned} h &= 8\ 00 \\ h' &= 18.24 \\ h'' &= 31.83 \\ h''' &= 50.00 \end{aligned}$$

Whence we obtain the following discharges

$$Q = 12\ 56\sqrt{\frac{32}{0.31}} = 127.6$$

$$q = 3.14\sqrt{\frac{8}{0.32}} = \underline{15.7}$$

$$Q + q = 143.3$$

$$Q' = 12.56\sqrt{\frac{40.96}{0.31}} = 144.4$$

$$q' = 3.14\sqrt{\frac{12.48}{0.32}} = \underline{19.6}$$

$$Q' + q' = 164.0$$

$$Q'' = 12.56\sqrt{\frac{54.36}{0.31}} = 166.3$$

$$q'' = 3.14\sqrt{\frac{15.66}{0.32}} = \underline{22.0}$$

$$Q'' + q'' = 188.3$$

$$Q''' = 12.56\sqrt{\frac{72.68}{0.31}} \mp 192.3$$

This shows a pretty fair agreement between the volumes discharged, the discrepancies being due to the fact that our assumed

value of h was not sufficiently close for a fine calculation. The figures are near enough, however, to serve the purpose of showing to how small an extent, comparatively, the results are changed by the very considerable changes made in the inclination of the branch pipes. Later on we shall have occasion to notice more fully the small relative changes made in the volumes discharged through given pipes by changes of grade: for the present we will only call attention to the slight variations produced in the hydraulic grade line, as determined by the piezometric heads.

CHAPTER III.

Numerical example of a system of pipes for the supply of a town—Establishment of additional formulæ for facilitating such calculations—Determinations of diameters—Pumping and reservoirs—Caution regarding calculated results—Useful approximate formulæ—Table of 5th powers—Preponderating influence of diameter over grade illustrated by example—Maximum velocities. (Note.)

As a further study of a system of pipes to deliver water, let us suppose a town divided by intersecting streets into blocks 1,000 ft. sq., as shown in Fig. 9. We will suppose that the proposed water supply requires a total volume of 3 cu. ft. per second, equal to say 800,000 U. S. galls. in 10 hours.

The water is to be introduced by a central main $A\ B\ C$, and delivered east and west by the side mains $D\ D'$, $E\ E'$, $F\ F'$, $G\ G'$, $H\ H'$. At the extremities of these mains, the water is to be delivered at the elevations above datum indicated by the figures placed in brackets. The side mains $D\ D'$ and $E\ E'$ are to deliver each, east and west, $\frac{1}{4}$ cu. ft. per second, which quantity we will suppose is to be carried through the whole length of the pipe and delivered at its extremity at the maximum elevation, without regard to the quantities drawn off *en route* by the service pipes and smaller north and south mains, nor those drawn off by the lower taps. This will secure a good delivery of water in case of fires. The total delivery of the above two side mains will therefore be 1 cu. ft. per second. The remaining three side mains, $F\ F'$, $G\ G'$, and $H\ H'$, are to deliver, similarly, $\frac{1}{3}$ cu. ft. per second at each extremity, making 2 cu. ft. for the three.

These being the data, we will suppose the problem to be the

determining of the respective diameters of the pipes, and the height to which the water must be raised in a supply reservoir or standpipe, situated somewhere to the north of the town.

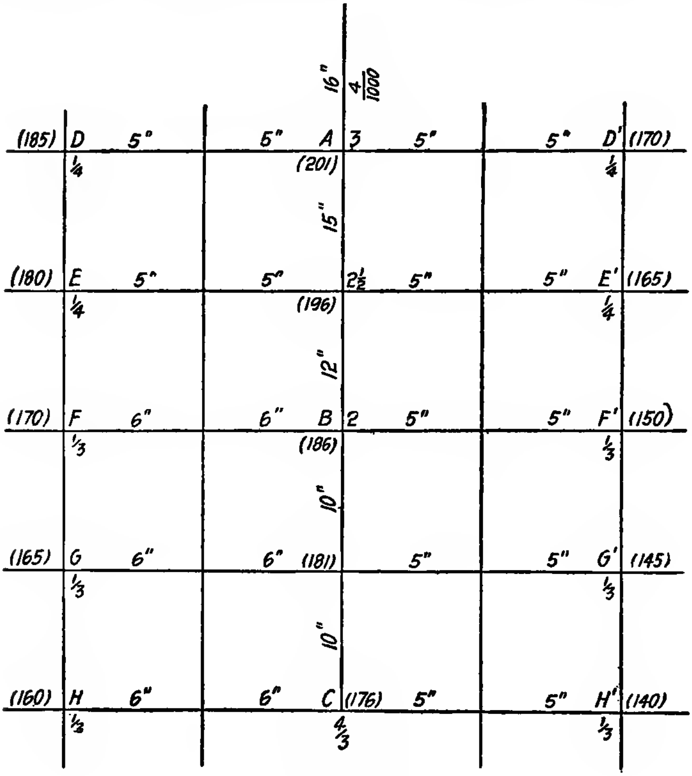

FIG. 9a.

The problem thus stated is indeterminate and admits of an indefinite number of solutions, for we may either use large pipes and low elevations, or small pipes and high elevations. Practically, however, there are limitations to this; for in the first place we shall naturally be restricted as to the height to

which it would be possible or advisable to raise the water, and secondly, experience shows that we should confine ourselves within certain limits as regards the velocity of the water in the pipes.

Generally speaking, these velocities should not exceed such as would be produced by a fall of from 4 to 8 ft. per thousand, according to the size of the pipe; the greater fall belonging to the smaller diameter. (*See note at end of chapter.*)

Before commencing the calculations, it will be well to establish certain additional formulæ, derived from (4), which are frequently of considerable utility.

When the length and diameter are constant:

$$\frac{Q'^2}{Q^2} = \frac{H'}{H}$$

When the head and diameter are constant:

$$\frac{Q'^2}{Q^2} = \frac{L}{L'}$$

When the head and length are constant:

$$\frac{Q'^2}{Q^2} = \frac{D'^5\, C}{D^5\, C'}$$

$$\frac{D'^5}{D^5} = \frac{Q'^2\, C'}{Q^2\, C}$$

When the head and discharge are constant:

$$\frac{D'^5}{D^5} = \frac{L'\, C'}{L\, C}$$

When the length and discharge are constant:

$$\frac{D'^5}{D^5} = \frac{H\, C'}{H'\, C}$$

These relations indicate that, other things being equal, the squares of the discharges vary directly as the heads and the fifth powers of the diameters, and inversely as the lengths; and that, other things being equal, the fifth powers of the diameters vary directly as the squares of the discharges and the lengths, and inversely as the heads.

As these relations are generally used for approximations, the coefficients may be dropped, and the equations written in this form :

$$Q' = \sqrt{\frac{Q^2 \times H'}{H}} \qquad (6)$$

$$Q' = \sqrt{\frac{Q^2 \times L}{L'}} \qquad (7)$$

$$Q' = \sqrt{\frac{Q^2 \times D'^5}{D^5}} \qquad (8)$$

$$D' = \sqrt[5]{\frac{D^5 \times Q'^2}{Q^2}} \qquad (9)$$

$$D' = \sqrt[5]{\frac{D^5 \times L'}{L}} \qquad (10)$$

$$D' = \sqrt[5]{\frac{D^5 \times H}{H'}} \qquad (11)$$

Other combinations can be made from these relations. Thus :

$$D' = \sqrt[5]{\frac{D^5 \times H \times Q'^2}{H' \times Q^2}} \qquad (12)$$

Commencing now with the west side of the main $H\,H'$, we have $\frac{1}{3}$ cu. ft. to be delivered at an elevation of (160) above datum. As the pipe will be a comparatively small one, we will assume a grade of $\frac{8}{1000}$, which will give a rise of 16 ft. between the extremity and the main junction, and requires an elevation of piezometric head, at this junction, of (176), as shown in the figure.

To obtain the proper diameter of pipe for this grade and discharge, we have, using (4), and assuming $C = 0.00076$ as a probable value ;

$$D^5 = \sqrt{\frac{(\frac{1}{3})^2 \times 1000 \times 0.00076}{8 \times 0.61}}$$

whence $D^5 = 0.017304$
and $D = 0.444$.

Or, for the next highest even inch :

$$D = 6 \text{ inches},$$

As regards the diameter of the pipe on the east side, since the length and discharge are the same as for the west side, and only the heads vary, being respectively 16 and 36 ft., it can be obtained by means of (11).

Thus :

$$D' = \sqrt[5]{\frac{0.017304 \times 16}{36}}$$

$$D' = 0.3777$$

or, for next highest even inch :

$$D' = 5 \text{ inches}.$$

The above head of 18 ft. per thousand produces a velocity of flow in a 5 in. pipe of a little over 3 ft. per second, which is somewhat greater than it should be. If the limit of velocity is overstepped to any considerable degree in a system of pipes such as we are considering, it would be best to use a larger pipe and check its flow down to the desired delivery by means of a gate or stop cock placed near its upper end, the effect of which will be to diminish the head. In the present instance the excess of velocity is probably not sufficient to render this precaution necessary.

The elevations are such that the above diameters of 6 and 5 ins. are also proper for the side mains $G\ G'$, $F\ F'$.

It is now necessary to calculate the diameter of the central main from B to C. This main might be divided into two parts, that between $F\ F'$ and $G\ G'$ and that between $G\ G'$ and $H\ H'$, but we will calculate it upon the supposition of a uniform diameter, capable of delivering the entire volume of $\frac{3}{4}$ cu. ft. per second as far as $H\ H'$.

4/3 = ?

Assuming a probable value of $C = 0.00066$, we have from (4):

$$D^5 = \frac{\frac{16}{9} \times 1.32}{6.1}$$

whence:

$$D^5 = 0.3847$$

and:

$$D = 0.826 = 10 \text{ ins.}$$

Taking now the mains $E\ E'$ and $D\ D'$, and beginning on the west side, assuming as before a grade of 8 ft. per 1,000, we find the length and head equal to those of $F\ F'$ etc., the only difference being the quantity it is desired to deliver, which is now $\frac{1}{4}$ cu. ft. as against $\frac{1}{3}$ in $F\ F'$. The relation (9) is therefore applicable, and we have:

$$D = \sqrt[5]{\frac{0.017304 \times \frac{1}{16}}{\frac{1}{9}}}$$

whence:

$$D^5 = 0.0097335$$

and

$$D' = 0.396$$

or, say,

$$D' = 5 \text{ ins.}$$

The mains on the east side are determined as before:

$$D' = \sqrt[5]{0.0097335 \times \frac{16}{31}}$$

$$D' = 0.346$$

This is not quite $4\frac{1}{4}$ ins., but to insure the desired delivery, it will be best to take the next highest even inch, and call it 5 ins.

As regards the central main from A to B, we find two grades, the upper one $\frac{5}{1000}$ and the lower $\frac{10}{1000}$. The lower section must deliver, under a grade of $\frac{10}{1000}$, all the water required for $F\ F'$, $G\ G'$, and $H\ H'$, aggregating 2 cu. ft. per second. Using (4), and taking 0.00066 as a probable value of C, we have:

$$D^5 = \frac{4 \times 0.66}{6.1}$$

whence :

$$D^5 = 0.4328$$

and :

$$D = 0.846$$

This is very nearly 10¼ ins., and a 10 in. pipe would answer, though 12 ins. would be better.

The upper section must deliver 2.5 cu. ft. per second, under a grade of $\frac{5}{1000}$. Taking the same probable value of C, we have :

$$D^5 = \frac{6.25 \times 0.66}{3.05} = 1.35 \quad (2)$$

whence :

$$D = 1.287 \qquad 1,062 \ (2)$$

which we can take as either 15 or 16 ins. 14″

This diameter might have been obtained from that of the lower section, by means of (12). Thus :

$$D'^5 = 0.4328 \times \frac{10}{5} \times \frac{6.25}{4} = 1.35 \quad (2)$$

$$D' = \cancel{1.287} = 1.062$$

This last formula might have been used throughout, but (4) is about as short and convenient ; frequently more so.

The diameters being thus determined, the quantities should be verified by (3). They will be found somewhat in excess of those proposed, owing to the general increase of the diameters.

As regards the height to which the water must be raised, the data show that 3 cu. ft. per second must be raised to a sufficient height to reach $D\,D'$ at an elevation of (201) above datum. If we adopt a grade of $\frac{4}{1000}$, the proper diameter of the pipe would be :

$$D^5 = \frac{9 \times 0.65}{2.44} = 2.39 \quad (2)$$

$$D = 1.32 \qquad 1.19$$

or,

$$D = 16 \text{ ins.}$$

If, instead of pumping, the water were collected in a reservoir by damming up the natural flow of some stream, and the dam were of necessity situated at an elevation so great that a danger-

ous pressure is apprehended, it would be necessary to first receive the water into a distributing reservoir situated at a lower level, or else, as a less advantageous expedient, to reduce the pressure by gates, properly located for the purpose.

It should be well understood that all the above assumed data, particularly such as relate to heads, are subjected to considerable variation in actual practice. All the calculations have been based, of necessity, upon the hypothesis that the exact allotted volume per second is being simultaneously drawn from the whole system, This would rarely be the case; for at any given second, the draught would be liable to fluctuate greatly from the average. Indeed, these calculations should only be regarded as fixing, with some degree of approximation, the proper relative discharges and pressures at the different points supplied.

The remaining north and south pipes should be calculated in the same way. Thus, those below $F F'$ on the west side discharge 1·6 cubic ft. with a grade of $\frac{5}{1000}$. This would require a 4-in. pipe. The draught from these would somewhat lower the piezometric heads at their junctions with the side mains. In a fine calculation, these reductions should be worked out, as was done in the previous example of branch pipes; in general, however, and in cases where the whole supply is supposed to be carried through to the extremity of the mains, and delivered at the highest elevation, as was done in the present instance, and where a liberal interpretation has been given to the calculation of diameters, this is not indispensable. At the same time, it should be a guiding principle of water-works engineering that a few hours spent in the office, in what may sometimes be considered an over-refinement of calculation, is by no means a waste of time, and frequently enables one to make advantageous and economical modifications in a project of distribution.

It may here be noted that (12) admits of being put into a very convenient form for rapid approximations. To do this, we

have only to calculate the discharge of a pipe 1 ft. in diameter, with a fall of 1 ft. per thousand, and to refer all other discharges with the fall per thousand feet to it, in order to obtain the corresponding diameter. The quantity discharged by the above pipe is 0.961 cu. ft. per second, and the square of the same is ~~0.924~~. Equation (12) may then be written :

1.61

$$D = \sqrt[5]{\frac{Q^2}{H} \times 1.08}$$

or very nearly :

$$D = \sqrt[5]{\frac{Q^2}{H}} \qquad (13)$$

we have also very nearly :

$$Q = \sqrt{D^5 \times H} \qquad (14)$$

which may be more conveniently expressed thus :

$$Q = D^2 \sqrt{D \times H} \qquad (14\ bis)$$

We have, also,

$$V = \sqrt{D \times H \times 1.6} \qquad (14\ ter.)$$

in which V = velocity in feet per second.

These last formulæ, it will be perceived, are based on the fact that, given a certain probable degree of roughness, a pipe 1 ft. in diameter, with a fall of 1 ft. in a thousand, will deliver 1 cu. ft. of water per second. If we desire to apply them to smooth, clean pipes, we have only to *halve the coefficient* for a 12-in. pipe, which will be equivalent to writing the above formulæ thus :

$$D = \sqrt[5]{\frac{Q^2}{2H}} \qquad (15)$$

$$Q = \sqrt{D^5 \times 2H} \qquad (16)$$

These formulæ will be found of very great utility in arriving quickly at approximate results. They can be advantageously used in sketching out a network of pipes such as we have just been considering. To facilitate their use the following table of fifth powers

has been calculated. This table indicates, by inspection, the diameters in inches corresponding to the fifth roots of the right-hand side of the equations, expressed in feet.

Diameters in inches.	*Fifth Powers in feet.*	*Diameters in inches.*	*Fifth Powers in feet.*
3	0.000977	22	20.72
4	0 004115	24	32.00
5	0.01256	26	47.75
6	0.03125	28	69.17
8	0.1317	30	97.66
10	0.4019	32	134 9
12	1.0000	34	182.6
14	2.1615	36	243.0
16	4.214	40	411.5
18	7.594	42	525.2
20	12.86	48	1,024 0

All the diameters which have been already calculated can be obtained very nearly by the use of (13). Relations (13) and (14) might also have been used in some of the previous examples.

Formulæ (13) and (14) serve to show the comparatively small influence of *grade* as affecting the volumes discharged, which point has been already alluded to, and the preponderating influence of *diameter*. Thus, we see by the above formulæ, that for a diameter of 1 ft. and a fall of $\frac{1}{1000}$, the volume of discharge is 1 cu. ft. If we wish to double this discharge by increasing the fall, we must adopt a grade of $\frac{4}{1000}$, i. e., we must quadruple the fall. If, on the other hand, we wish to produce the same result by increasing the diameter without changing the grade, we need only adopt a diameter of 1.32 ft. and even a little less, on account of the decrease in the coefficient. That is to say, to double the discharge, we must increase the fall 300 per cent., or the diameter 32 per cent.

NOTE.—In completion of what has been already said in this chapter (page 37), regarding the limit of velocities for pipes of different diameters, the following table (founded upon that given by Mr. Fanning) indicates pretty closely the maximum velocities which it is generally advisable to produce :

Diameter in inches..........	6	12	18	24	30	36	42	48
Velocity in ft. per sec.......	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

CHAPTER IV.

Use of formula (14) illustrated by numerical example of compound system combined with branches—Comparison of results—Rough and smooth pipes—Pipes communicating with three reservoirs—Numerical examples under varying conditions—Loss of head from other causes than friction—Velocity, entrance and exit heads—Numerical examples and general formulæ—Downward discharge through a vertical pipe—Other minor losses of head—Abrupt changes of diameter—Partially opened valve—Branches and bends—Certrifugal force—Small importance of all losses of head except frictional in the case of long pipes—All such covered by "even inches" in the diameter.

As an illustration of the use of (14) we will calculate by its aid the discharge from a reservoir, tapped at a depth of 50 ft. by a horizontal compound system consisting successively of 2,000 ft. of 12-in. pipe, 2,000 ft. of 24-in. pipe and 2,000 ft. of 12-in. Each of these three lengths of pipe is tapped midway by a 6-in. pipe, laid horizontally, the one nearest the reservoir having a length of 3,000 ft.; the next 1,000 ft., and the last 500 ft. (See Fig. 9, *bis.*) All the pipes being open, it is desired to find the piezometric heads h, h', h'', h''', h'''', at each branch and change of diameter, and the volumes discharged by each branch and section of main pipe.

Beginning at the lower end and assuming 6 ft. as an approximate value of h, we have from (14), H always representing the fall per 1,000 :

$$\sqrt{6} + \sqrt{\frac{12}{32}} = \sqrt{h' - 6}$$

$$h' = 15.36$$

$$\sqrt{9.36} = \sqrt{32\,(h'' - 15.36)}$$

$$h'' = 15.65$$

$$\sqrt{9.36} + \sqrt{\frac{15.65}{32}} = \sqrt{32\,(h''' - 15.65)}$$

$$h''' = 16.09$$

$$\sqrt{14.08} = \sqrt{h'''' - 16.09}$$
$$h'''' = 30.17$$

$$\sqrt{14.08} + \sqrt{\frac{10.06}{32}} = \sqrt{h''''' - 30.17}$$
$$h''''' = 48.82$$

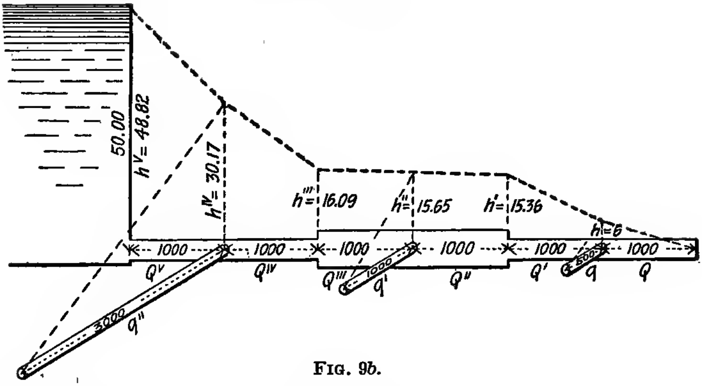

FIG. 9*b*.

Comparing this value with the given height 50, we may increase all the preceding values of h, h', etc., in the proportion of $\frac{50}{48\ 82}$. But in practice we would not wish to reckon on the total head, and it would be preferable therefore to let the values stand as they are.

We will now calculate the quantities, calling those discharged from the successive sections of main pipe, beginning at the lower end, Q, Q', Q'', Q''', Q'''', and Q''''', and those discharged by the branches, beginning also at the lower end, q, q', q'', respectively, using both (3) and (14). The results given by (14) naturally check exactly, since they depend directly upon the method used in determining h, h', etc.

	By (3)	By (14)
Q	= 2.39	2.45
q	= .56	.61
$Q + q$	= 2.95	3.06

	By (3).	By (14).
Q'	$= 2.96$	3.06
Q''	$= 2.99$	3.05
q'	$= .65$	.70
$Q'' + q'$	$= 3.64$	3.75
Q'''	$= 3.68$	3.75
Q''''	$= 3.63$	3.75
q''	$= .52$	.56
$Q'''' + q''$	$= 4.15$	4.31
Q'''''	$= 4.18$	4.32

The above example was very favorable to the use of (14), because of the lengths assumed for the different pipes, but in almost all cases it will greatly reduce the volume of calculation, and frequently give sufficiently close results. Indeed, as all these calculations are merely approximations, and as we have taken our coefficients pretty high, it would no doubt often be found, could the actual discharges be measured, that the apparently less exact formula gave the more correct results.

In all the previous examples, the coefficients for rough pipes have been used. It is well to remember that, as is shown by (15) and (16), the discharge of a clean pipe of given diameter is about 41 per cent. greater than that of a rough pipe of the same diameter; also that the diameter of a clean pipe, discharging an equal volume with a rough one, will be about 88 per cent. of the latter. Between these limits of smoothness and roughness there are, of course, an indefinite number of gradations.

A very interesting investigation is that of a system of pipes communicating with two reservoirs, and discharging either freely in the air, or into a third reservoir situated at a lower elevation as shown in Fig. 10.

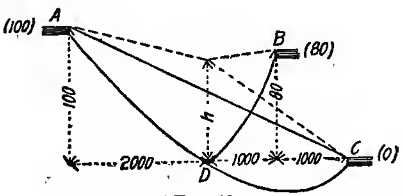

FIG. 10.

Let us suppose the water surfaces in A and B to be respectively 100 and 80 ft. above the water surface in C, and that all the pipes shown in the figure are 12 ins. in diameter. Let the total length of pipe from A to C be 4,000 ft.

If communication were shut off from B, the flow would be direct from A to C; if communication were shut off from C, it would be direct from A to B. If A were shut off, the flow would be from B to C. If all the communications were wide open, we desire to know whether the flow would be from A to B and C, or from A and B to C; and in either case, to know the piezometric head h, at the junction D, and the volumes discharged.

First, let the junction D be situated midway in the 4.000-ft. pipe joining A and C, and let the length $B\,D$ be 1,000 ft. Let us for a moment revert to the supposition that B is shut off. The flow would then be from A to C, the hydraulic grade line would be a straight line joining the surfaces A and C, and under our present hypothesis, that the junction D is in the middle of $A\,C$, the piezometric head h would be 50 ft. above the surface of the lower reservoir C. But B is supposed to be 80 ft. above the same, and therefore the flow must be from A and B to C. We might at first sight suppose that the flow from B to C would be in virtue of the head $80 - 50 = 30$ ft., which is the difference of level between B and the piezometric head at the junction; but just as a branch drawing water *from* a main pipe lowers the piezometric head at the junction, so does a branch discharging *into* the main pipe raise it. It is necessary to see what the height h will be in the present case.

The quantity discharged into C is equal to the sum of the quantities passing from A and B. All areas and coefficients being equal, and all reductions made, we have :

$$\sqrt{\frac{h}{2}} = \sqrt{50 - \frac{h}{2}} + \sqrt{80 - h}$$

whence :

$$h = 65 + \sqrt{4000 - 90\,h + \frac{h^2}{2}}$$

and, by successive approximations :

$$h = 74$$

Using this value of h in (3), we obtain the different discharges as follows :

$$Q = 5.88$$
$$Q' = 3.48$$
$$Q'' = 2.37$$

This gives a very close agreement in the relation $Q = Q' + Q''$.

Suppose now that the diameter of the branch $B\ D$ be reduced to 6 in., all the other conditions remaining the same. Still regarding the coefficients as equal, in order to get rapidly at an approximation, factoring the areas and simplifying, we have :

$$4\sqrt{\frac{h}{2}} = 4\sqrt{50 - \frac{h}{2}} + \sqrt{40 - \frac{h}{2}}$$

whence :

$$16.5\,h = 840 + 4\sqrt{8000 - 180\,h + h^2}$$

and, by successive approximations :

$$h = 58$$

This value of h gives the following quantities :

$$Q = 5.21$$
$$Q' = 4.43$$
$$Q'' = 1.08$$

A tolerably close check, but showing that the true value of h is a little greater than the even 58 ft. at which we have placed it.

Let us now suppose that the pipe $B\ D$ is increased to a diameter of 36 in., all the other conditions remaining as before.

Then :

$$\sqrt{\frac{h}{2}} = \sqrt{50 - \frac{h}{2}} + 9\sqrt{80 - h}$$ -2,+3

whence :

$$h = 79.90$$

Giving :

$$Q = 6.111$$
$$Q' = 3.065$$
$$Q'' = 2.816$$

a close approximation; the true value of h lies between 79.85 and 79.90.

As h increases with the diameter of the pipe $B\ D$, it might at first seem as though, by indefinitely increasing the diameter, h might be so increased as to cause a flow from A into B. A moment's reflection, however, will show that under the assumed conditions the diameter can never be sufficiently increased to cause a flow toward B. For it has been seen that when B is shut off, the piezometric head at D is 50 ft. It is raised by opening the communication with B, and allowing water to flow into the main from B. It is evidently, therefore, an essential condition of the increase of piezometric height that the flow should be from, not to, the reservoir B.

But the effect will be different if the junction D be sufficiently advanced toward the reservoir A. Let us suppose the positions of the three reservoirs to remain the same, all the pipe diameters to be 12 ins., and the point of junction of the pipe $B\ D$ to be placed at 500 ft. from A (Fig. 11). If communication with B were shut off, the piezometric height at D would be 87.5 ft. There would therefore be a flow from A to B and C when the pipe leading to B was open. But this flow would not take place under the head 87.5, for the draft toward B would lower it.

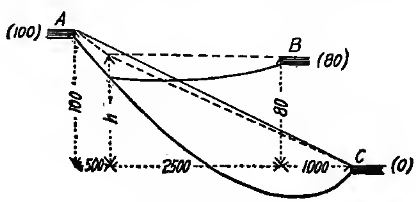

FIG. 11.

To ascertain the true value of h at the point D, we have the relation:

$$\sqrt{\frac{100-h}{500}} = \sqrt{\frac{h}{3500}} + \sqrt{\frac{h-80}{2500}}$$

simplifying:

$$\sqrt{100 - h} = \sqrt{\frac{h}{7}} + \sqrt{\frac{h - 80}{5}}$$

$$47\,h = 4060 - 11.86\ \sqrt{h^2 - 80\,h}$$

whence, by successive approximations:

$$h = 82.65$$

Using this value of h we get:

$$Q = 5.695$$
$$Q' = 4.698$$
$$Q'' = 0.995$$

When B is shut off, in the above system, the discharge from A to C is 4.33 cu. ft. per second.

In all that precedes, only the resistance due to friction has been considered, and the total difference of level between the source of supply and the discharge has been taken as available for overcoming this frictional resistance. In the case of long pipes, where the velocity is comparatively low, this resistance is so greatly in excess of all the others that, in order to simplify calculations, they are neglected. This leads to no material error in cases where the pipe is over 1,000 diameters in length.

Attention, however, has been already called to the fact that there are other resistances which require a certain proportion of the total head to overcome them, leaving only the remainder available as against friction. Indeed, it is evident, if we assume all the head to be consumed by frictional resistance alone, the water in the pipe would be in exact equilibrium, and no flow could take place.

It will now be proper to show how the total loss of head, from all causes, may be calculated. And first, a word in reference to the phrase "loss of head" just employed. This term, often met with in treatises on hydraulics, may occasionally prove confusing. It is really little more than a convenient abbreviation. When we speak, for instance, of "the loss of head due to velocity," we mean the head, or fall, theoretically necessary to

produce that velocity. Similarly, when we speak of "the loss of head due to resistance to entry," we mean the amount of head, or pressure, necessary to force the fluid vein into the mouth of the pipe or orifice, against the resistance of its edges. This resistance, it may be remarked in passing, as well as that due to bends, elbows and branches, shortly to be mentioned, is caused by the fact that water is not a perfect fluid, and therefore changes of direction in its flow require a certain amount of force to break or distort the form of the fluid vein as, though to a very much less degree, would be the case with a plastic body under similar circumstances. The property of water which causes these resistances is called its *viscosity.*

As applied to long pipes, the principal "loss of head," and the only one hitherto considered, is the *frictional.* The term thus applied means the height or pressure necessary to overcome the friction of the water passing with a given velocity through a pipe of given length and diameter. Thus, when we speak of the frictional loss of head per 1,000 ft. in reference to a given pipe, we mean the fall per 1,000 ft. necessary to maintain the given or desired velocity, as against friction.

We will now investigate this subject by means of the following problem: Two reservoirs (Fig. 12) containing still water and having a difference of level of 30 ft., are joined by a pipe 12 ins. in diameter and 3,000 ft. long. What is the velocity of discharge between the upper and lower reservoirs?

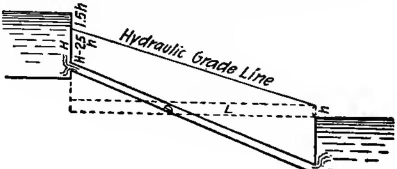

FIG. 12.

From what has been already said, it will be seen that, besides the frictional loss of head, there will be the loss of head due to

velocity and that due to entrance. If the pipe discharged freely in the air at its lower end, at the vertical distance of 30 ft. below the surface of the water in the upper reservoir, these three would be the only losses of head incurred, and their sum would be equal to 30 ft.; but as the discharge takes place in a reservoir, the surface of the water in which is supposed to cover the end of the pipe, to a sufficient depth to cause the discharge to take place in still water, there is the further loss of head due to the *extinction of the velocity* which is dissipated in vortices. This loss constituted what may be called the *back pressure* of the reservoir.

In solving this problem, let us first, as heretofore, neglect all losses except frictional ones. We have then, from (1), using the above data, and the coefficient for rough pipes :

$$\frac{1}{100} = 0.00066\ V^2$$
$$V^2 = 15.15$$
$$V = 3.89 \text{ ft. per second.}$$

The head theoretically necessary to produce this velocity is given by the formula derived from the law of falling bodies, $h = \frac{V^2}{2g}$ by substitution of the above value V. Thus :

$$h = \frac{15.15}{64.4}$$
$$h = 0.2352$$

Besides this, there is the loss of head due to entrance. We have already seen that this is always equal to about half the velocity head. We have then :

$$h + \frac{h}{2} = 0.3528$$

The loss of head from back pressure of the water in the lower reservoir, being that necessary to extinguish the velocity, must be equal to that necessary to produce the same. We have, therefore, for the total losses, outside of friction :

$$h + \frac{h}{2} + h = 0.588$$

And the head available for overcoming friction becomes

$$30 - 0.588 = 29.412$$

We must now recast our original calculation, using 29.4 ft. instead of 30 as available frictional head. Thus:

$$\frac{29.4}{3000} = 0.00066\,V^2$$
$$V^2 = 14.8$$
$$V = 3.85$$

This is a very small reduction from the velocity already obtained. But, in order to see how our previous solution is affected by the change, we will work out new values for the subheads. Thus:

$$h = \frac{14.8}{64.4}$$
$$h = 0.23$$
$$h + \frac{h}{2} + h = 0.575$$
$$30 - 0.575 = 29.425,$$

leaving the previous value practically unchanged.

Let us now see, by means of a general formula, what is the amount of error which we commit when we ignore all resistances except friction.

Calling V the actual mean velocity, that is the actual volume discharged divided by the area of the pipe (3), we have, in the case of discharge between two reservoirs, as shown in Fig. 12, the following subheads, which together make up the total head H:

$$H = \frac{V^2}{2g} + \frac{V^2}{4g} + \frac{V^2}{2g} + \frac{L\,C\,V^2}{D}$$
$$H = \frac{5\,V^2}{4g} + \frac{L\,C\,V^2}{D}$$
$$H = 0.039\,V^2 + \frac{L\,C\,V^2}{D}$$

That is to say, by using (3), which gives

$$H = \frac{L\,C\,V^2}{D}$$

we make the error of omitting a height not quite equal to 4 per cent. of the square of the velocity.

In long pipes this is a very trifling amount.

If the pipe discharged in free air, we would have :

$$H = \frac{V^2}{2g} + \frac{V^2}{4g} + \frac{L\,C\,V^2}{D}$$

$$H = 0.0233\,V^2 + \frac{L\,C\,V^2}{D}$$

In this case we make the still smaller error of omitting $2\frac{1}{3}\%$ of V^2.

In all cases, having obtained V^2 by means of (1), we can easily judge from the nature of the problem whether it is necessary to take account of these errors. In designing a system of pipes, where the problem generally is to find the proper diameter for a certain discharge, the practice of taking the next highest even inch will almost always amply suffice to cover all omissions.

As has been already stated, in all ordinary circumstances of pipelaying, the horizontal measurement of the pipe is taken instead of its actual length. It is only in special cases that this cannot be done. The extreme limit occurs in the case of a vertical pipe discharging from the bottom of a reservoir. This constitutes a very interesting special case, for should the reservoir be of indefinitely large area, but of relatively shallow depth, the relation $\frac{H}{L}$ tends toward unity as L, and consequently H, increase. The velocity, as determined by (1), tends therefore toward :

$$V = \sqrt{\frac{D}{C}}$$

and remains constant, no matter how greatly L may be increased. If we apply this formula to a 12-in. pipe of indefinite length, using the coefficient for rough pipes, we get,

$$V = 38.9$$

This is the maximum velocity of discharge in feet per second for a vertical 12-in. pipe under the given circumstances.*

There are several minor losses of head, besides those already considered, which are liable to occur from changes of diameter, branches, and bends or elbows. Our experimental knowledge of the effects of these features is very limited, and it is probable that much weight should not be attached to the formulæ given for their determination. A brief space will be devoted to their consideration, more with a view to make the present paper complete than for any practical value which they possess.

When water passes through a pipe of which the diameter is abruptly changed, at a certain point, to a greater or a smaller one, there is a loss of head due to the eddies formed and the sudden contraction of the fluid vein. In practice such pipes are always joined by a *reducer,* or special casting, which forms a tapering connection between the two. This greatly diminishes the agitation of the water in passing from one pipe to the other. It would seem, however, that the mere change of velocity, independent of such agitation, causes some slight modification of the profile of the hydraulic grade line; and it will be well, in any event, to give formulæ for the different cases which may occur when abrupt changes take place, as these give rise to the maximum retardation. The following formulæ are taken from Claudel's *Aide Mémoire,* ninth edition.

First.—When the change is from one pipe to another of smaller diameter, we have:

$$h = 0.49 \frac{V^2}{2g}$$

whence:

$$h = 0.00076\ V^2$$

V being the velocity of the water in the smaller pipe. We have seen, by examples previously given, how this velocity may be obtained.

* The same result may be inferred from what has been said in Chapter I. about a pipe laid so that its axis coincides with the hydraulic grade line. Obviously, a vertical pipe discharging downward is a special case of such coincidence.

Second.—If the water (Fig. 13), in its passage from the greater to the smaller pipe, passes through an opening in a thin diaphragm,

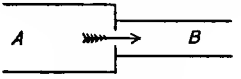

FIG. 13.

as in the case of a partially opened stop-cock, we have :

$$h = \frac{V^2}{2g} \cdot \left(\frac{S}{0.62\,S'} - 1 \right)^2$$

in which V is the velocity in B, S the area of cross-section of B, and S' the area of the opening in the diaphragm.

Third.—When the flow is from one pipe to another of larger diameter :

$$h = \frac{(V - V')^2}{2g}$$

in which V = velocity in small pipe, and V' = velocity in larger one. When the water passes from a pipe into a reservoir, as in the case lately considered, V' becomes zero, and we have, as already established in that case :

$$h = \frac{V^2}{2g}$$

Another loss of head is that due to branches (Fig. 14). In

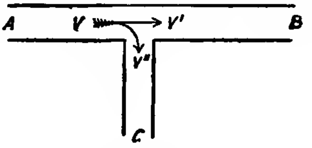

FIG. 14.

this case the water flowing from A, with a velocity V, is split at the junction, part passing on toward B, with a reduced velocity V', and part entering the branch and flowing toward C, with the velocity V''. The loss of head occasioned by perturbations of the water at the junction has not been satisfactorily investigated.

When the branch leaves the main at a right angle, this loss, as determined by a few incomplete experiments, is :

$$h = \frac{3\,V''^2}{2\,g}$$

V'' being the velocity in the branch. We have already seen how this velocity may be calculated.

If, as is generally the case in practice, the branch is deflected gradually instead of forming an abrupt angle of 90°, the vortices are nearly annulled, and the only loss can be from the difference of the velocities in the three pipes. Thus for B and C, respectively, we have :

$$h = \left(\frac{V - V'}{2\,g}\right)^2$$

$$h' = \left(\frac{V - V''}{2\,g}\right)^2$$

For bends, or elbows, Navier's formula for loss of head is :

$$h = \frac{V^2}{2\,g}\left(0.0128 + 0.0186\,R\right)\frac{A}{R}$$

in which $V =$ velocity of flow, $R =$ the radius of the bend, taken along the axis of the pipe, and $A =$ the length of the bend, also measured along the axis.

It will readily be seen how very trifling the loss of head from this cause will be in all ordinary cases.

The water passing around a bend exercises a radial thrust upon it which may sometimes be so considerable as to require bracing against. The expression for the centrifugal force F is :

$$F = \frac{M\,V^2}{R}$$

in which $M =$ the mass of the liquid in motion, $V =$ its velocity, and $R =$ the radius of the bend measured on its axis.

As an illustration, we will suppose a pipe 24 ins. in diameter, through which the water flows with the velocity of 8 ft. per second, around a bend of 8 ft. radius.

The mass of the liquid in motion is its weight divided by g. The centrifugal force, therefore, per running foot is :

$$F = \frac{3.14 \times 62.5}{32.2} \times \frac{8^2}{8}$$

$$F = 48.72 \text{ lbs.}$$

If the bend turns a quarter circumference, its development on the axis will be 12.57 ft., and the total thrust on the bend will be $48.72 \times 12.57 = 612.4$ lbs.

This would be liable to be intensified by sudden changes in velocity, and if the bend is not well abutted, might tend to draw the joints.

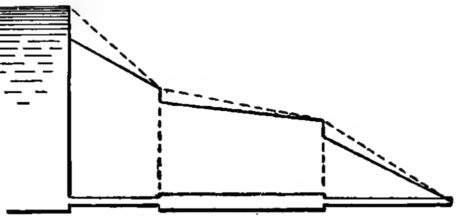

FIG. 15.

Fig. 15 shows the manner in which such losses of head as we have been just considering modify the the profile of the hydraulic grade line. The dotted line shows the grade as determined by the calculations which we have already made for a line of pipes of varying diameter. The full line, broken at the reservoir and at each change of diameter, shows the hydraulic grade as modified by losses of head due to velocity and changes of diameter. It will be understood, of course, that this is a mere random sketch, without reference to proportion.

The result of what precedes in reference to all losses of head other than friction shows that in practice, and in the case of long pipes, such losses exercise but a trifling influence. A very small increase in the diameter of the pipe over that obtained by calculation based on frictional head alone, such as would naturally be made to get even inches, will in almost all cases largely cover all losses due to velocity, entrance, branches, bends, etc.

CHAPTER V.

NOTES ON PIPELAYING.

It will not be amiss at present to give some hints respecting *Pipelaying*. Enough has been already said to show how greatly the smoothness or roughness of the interior of a pipe affects the velocity of the flow of water through it. A line of pipes is made up of a great number of separate lengths joined together, generally, by the *spigot end* of each pipe entering into the *hub* or *bell end* of the other. Each of these joints occasions more or less friction, and it is essential, not only on this account, but also and more particularly in order to make a substantial and enduring piece of work, that the pipes should be laid as evenly as possible, and the joints well fitted and calked. The alinement should be straight and the grade regular. This latter is the more important of the two, because sags and depressions in the line occasion deposits of impurities in the low points and accumulations of air in the high ones. The line should run straight and even between changes of grade and direction. Each low point, or point from which the grade rises both ways, should be provided with a special "blow-off" and stop cock, to clear it of sediment by blowing off under pressure, and each summit, or point from which the grade falls both ways, should have a special air vent, or hydrant, to discharge the accumulated air from time to time. When a new line of main is filled for the first time, or when a line is refilled after having been emptied for any cause, all the blow-offs and air cocks should be opened, and the water

admitted very slowly by giving a few turns only to the admission valve. Then, as the pipe gradually fills, the blow-offs should be closed progressively as the water reaches them, and the air cocks also, beginning to close the latter, if possible, from the lower end, and only when they discharge *solid water*. Changes of horizontal direction should be joined by as easy curves as can be obtained. In sharp curves and large diameters, special curved pipe may be necessary, but, in general, curves are got in with straight pipe, using all short pieces that may be on hand, and, if necessary, cutting whole pipe, and joining the straight pieces with sleeves.

In a well-laid pipe line, all pipe, particularly all those of 20″ diameter and upward, are laid on blocks. These blocks consist of pieces of wood sawed out to regular dimensions, there being two under each pipe, one just behind the hub and the other as near the spigot end as will permit of the joint being easily reached for calking, say about 2 feet. For diameters from 36″ to 48″, the length of these blocks may be eqnal to the diameter of the pipe, and about a foot wide and 6″ thick. For smaller pipe they may be about two feet longer than the diameter of pipe and proportionately lighter than for the larger sizes. The pipe is held in its place on these blocks by means of wooden wedges placed side by side, on opposite sides of the pipe, and driven past each other. For 48″ pipe these wedges may be about 18″ long, 6″ wide and 4″ thick at the bntt. For smaller pipe they will of course be lighter.

The instrumental alinement of the pipe line presents no particular difficulty, because the excavation once correctly started is not likely to deviate to any injurious extent. It is much more difficult, as it is more important, to keep the grade. This is best effected, practically, I think, in the following manner: Let the ordinary marked grade stakes be set for the excavation. Then when the proper depth has been reached, or nearly so, let grade plngs be driven in the bottom of the trench, every 50 feet or

oftener, with their heads exactly to grade. A line can then be stretched from one to the other, and the blocks laid to it. It is better to bed the blocks a trifle low, say a quarter to half an inch, particularly with heavy pipe and hard bottom, and then raise the pipe to grade by driving in the wedges. It is not necessary to set the pipes with rod and level. If the grade plugs have been driven as suggested, a competent foreman will adopt any one of many ways for sighting in the pipe to the proper level. With soft ground and heavy pipes, longitudinal stringers are advantageously employed under the blocks, the spaces between them being well packed with broken stone or other ballast.

When pipe have been laid and calked, it is advantageous to cover them as soon as possible by backfilling the trench, to prevent the joints from drawing in consequence of expansion and contraction due to exposure to the changes of temperature of the air.

In backfilling the trench after the pipe is laid, be very careful that the earth is well tamped in under the pipe, so that it may have a solid bearing throughout its entire length. The earth put in next to the pipe should be clean and free from stones. Be particularly careful that no large stone gets under the pipe, as in case of a sudden jar, such as would be produced by a casual "water hammer," it might punch a piece out of the pipe, or at least crack it.

In leading and calking joints, the specifications generally call for a certain depth of lead. The specifications of the city of New York require 4 ins. of lead for 48-in. pipe. It is a great advantage to have a deep joint, although the necessity is sometimes denied upon the ground that in calking it is impossible to "upset" the lead to a depth of more than perhaps half or ¾ of an inch, and that therefore the additional lead is of no advantage. This is, however, a mistake, for an abundant depth must be allowed for cutting off the protruding lead in recalking a joint which has drawn. The lead which is drawn out when a line of

pipe contracts is not forced back again when the pipe expands; on the contrary it remains out, and, if the pipe contracts again, a fresh amount will probably be again drawn out, all of which must be cut off when the joint is recalked.

In calking joints, yarn or gasket is first driven in until the proper depth is secured, and lead is then poured in, either by the use of the clay dam or "snake," or by the use of a metallic "clip." The latter is very useful in laying large pipe, as it enable the whole joint to be run at one pouring. A refinement in running joints is to employ a lead gasket, for example a piece of lead pipe, hammered flat, and circled around the pipe. It is driven in and calked, and the remainder of the joint is then run and calked in the usual manner. The advantages of this system are that there is no perishable material used, and that the joint is calked on both faces, which is very favorable to making it tight. It is sometimes difficult to get the trench and pipe sufficiently dry to admit of pouring a molten joint. In such cases it is necessary to put the lead in cold, perhaps in the form of a ring of flattened lead pipe, as above, and calk as put in. This makes a very satisfactory joint, only it is slower and more expensive.

Mr. Billings, in his excellent treatise, "Some Details of Water-Works Construction," gives Mr. Dexter Brackett's formula for the average weight of lead in a joint about $2\frac{1}{2}$ in. deep, as follows:

$$l = 2d,$$

in which l = pounds of lead per joint, and d = inside diameter of pipe in inches. As the pipes are usually 12 ft. in length, the weight of lead per running foot equals one-sixth of diameter of pipe in inches. If a 4-in. joint is used, we would have

$$l = 3.2d,$$

and the weight per running foot under above assumption of length of pipe would be $\dfrac{d}{3.75}$, or, in round numbers, $\dfrac{d}{4.}$

APPENDIX.

As it is convenient in making estimates to have the correct weight of cast iron pipes of different diameters in handy form, I subjoin a table of weights of pipes made by the Warren Foundry, of Phillipsburg, N. J.:

TABLE SHOWING THICKNESS OF METAL AND WEIGHT PER LENGTH FOR DIFFERENT SIZES OF PIPE UNDER VARIOUS HEADS OF WATER.

Inside diam., in inches.	50 ft. head.		100 ft. head.		150 ft. head.		200 ft. head.		250 ft. head.		300 ft. head.	
	Thickness of metal.	Weight per length.	Thickness of metal.	Weight per length.	Thickness of metal.	Weight per length.	Thickness of metal.	Weight per length.	Thickness of metal.	Weight per length.	Thickness of metal.	Weight psr length.
2	.294	63	.312	67½	.330	72	.348	76½	.366	81	.384	86
3	.314	144	.353	149	.362	153	.371	157	.380	161	.390	166
4	.361	197	.373	204	.385	211	.397	218	.409	226	.421	235
5	.378	254	.393	265	.408	275	.423	286	.438	298	.453	309
6	.393	315	.411	330	.429	345	.447	361	.465	377	.483	393
8	.422	445	.450	475	.474	502	.498	529	.522	557	.546	584
10	.459	600	.489	641	.519	682	.549	723	.579	766	.609	808
12	.491	768	.527	826	.563	885	.599	944	.635	1,004	.671	1,064
14	.524	952	.566	1,031	.608	1,111	.650	1,191	.692	1,272	.734	1,352
16	.580	1,215	.604	1,253	.652	1,360	.700	1,463	.748	1,568	.796	1,673
18	.589	1,370	.643	1,500	.697	1,630	.751	1,761	.805	1,894	.859	2,026
20	.622	1,603	.682	1,763	.742	1,924	.802	2,086	.862	2,248	.922	2,412
24	.687	2,120	.759	2,349	.831	2,580	.903	2,811	.975	3,045	1.047	3,279
30	.785	3,020	.875	3,376	.965	3,735	1.055	4,095	1.145	4,458	1.235	4,822
36	.882	4,070	.990	4,581	1.098	5,096	1.206	5,613	1.314	6,133	1.422	6,656
42	.980	5,265	1.106	5,958	1.232	6.657	1.358	7,360	1.484	8,070	1.610	8,804
48	1 078	6,616	1.222	7,521	1.366	8,431	1.510	9,340	1.654	10,269	1.798	11,195

All pipe cast vertically in dry sand, in lengths of 12 ft., except the 2-in., which are cast 9 ft. long.

The general formula for weight of cylindrical cast iron pipe of given thickness is:

$$W = 0.82\,(D + T)\,T \times L, \qquad (1)$$

in which

W = weight in pounds.
D = inside diameter in inches.
T = thickness of metal in inches.
L = length in inches.

A convenient approximate formula for the weight per foot of cylindrical part of a cast iron pipe is:

$$W = 10\,(D + T)\,T. \tag{2}$$

In calculating the weight of cast iron pipe, they are always considered as being cylindrical, and eight inches of length are added as the equivalent of the hub or bell for all diameters. Thus a pipe measuring 12 feet over all, including the hub, would be considered, as regards weight, as a plain cylindrical pipe 12 feet 8 inches long. For instance, to calculate the weight of a 42-inch pipe, 12 feet over all and 0.980 inch thick, by the above approximate formula (2) we have:

$$W = 10 \times 12.67 \times 42.98 \times 0.98 = 5336.6 \text{ lbs.}$$

This is about $1\frac{1}{3}\%$ in excess of weight given in Warren Foundry table.

If we wish to find the thickness, the length, diameter and weight being given, we have from (1):

$$T = \sqrt{\frac{W}{0.82L} + \frac{D^2}{4}} - \frac{D}{2} \tag{3}$$

To find the proper thickness in inches corresponding to the head in feet. H, of pressure for a given diameter in inches, D, we have:

$$T = 0.00006\,H\,D + 0.0133\,D + 0.296.$$

The hydraulic engineer will find it both interesting and useful to work out concise formulæ covering frequently recurring cases, and enter them in his notebook for future reference. These should be carefully checked by testing them numerically, so that they can be used with confidence when wanted. For instance, it is a common practice to allow 100 United States gallons per capita to be consumed in 10 hours, in calculating the proper supply for a town. To find the diameter of a pipe to convey this amount we may use the approximate formula:

$$D = \sqrt[5]{\frac{(0{,}0004 \times N)^2}{H}}.$$

in which

D = diameter in feet.
H = fall per 1,000.
N = number of population to be supplied.

Also, in estimating theoretical horse power necessary to raise a given volume of water to a certain height and in a certain time, we have

$$\text{HP.} = \frac{Q \times H}{8.82}$$

and

$$\text{HP.} = \frac{Q' \times H}{5.7},$$

in which

Q = cubic feet per second.
Q' = millions of U. S. gallons per 24 hours.
H = lift in feet.

In calculating the power necessary to pump water into a reservoir situated at certain horizontal and vertical distances from the pumps, and connected with them by a force main of given diameter, we must imagine the water to be raised to a certain elevation above the reservoir such that the difference of level between this elevation and that of the reservoir shall be sufficient to convey the required amount of water to the reservoir against the friction of the force main. Thus, suppose we wished to deliver 1 cubic foot per second through a 12-inch force main to a reservoir 100 feet above the pumps, and distant 10,000 feet from them. By our approximate formula (13) we know that this requires a fall of one foot in a thousand, so, as the distance is 10,000 feet, we need 10 feet to overcome the friction, and our pumps must therefore be able to raise the given volume a total height of 110 feet. This calls for 12.47 theoretical horse power, by the formula just given.

SECOND PART.

NOTES ON WATER SUPPLY ENGINEERING.

QUALITY OF WATER.

The first question in regard to a water supply is, evidently, *quality*. The solution of this question, either generally or in any particular case, is by no means a simple nor any easy one, particularly since it is now fully recognized that chemical analysis, to which we naturally recur in such cases, is far from being a final criterion, but is only one element in a group of data from which we infer the probable quality of a given source of supply.

As animal refuse and the waste products of human industry are the principal sources of menace to a water supply, we commonly look for a high degree of purity in water drawn from thinly populated districts devoid of manufactories. Such districts, however, are not often to be met with in the vicinity of large towns, and, even when they are, we must expect a gradual encroachment upon them, from the natural growth of the neighborhood.

One excellent criterion of quality is the general health of the communities using the water. Also, the order of fishes which find their habitat in the streams of a given district. Little danger, for instance, would be apprehended from the use of a good trout stream.

Although all the fresh water used upon the earth reaches it

in the form of rain which is first drawn up by evaporation, chiefly from the sea, there are several different forms under which it presents itself for our use. The first broad classification of these forms would be that which divides the supply into *surface water* and *ground water*. By surface water is understood the water of lakes, ponds, rivers and streams, all water which in fact is collected directly from the surface of the earth; and by ground water, that which is derived from wells and filtering galleries, and from springs when taken at or near their source.

Each of these classes admits of much subdivision, but the differences will be principally those of degree, and not of kind.

For instance, we have the smaller streams, such as creeks, brooks, etc., and also the larger ones, rising to the dignity of rivers. While these certainly do present slightly different characters, still their main difference is one of size. Again, though the waters of lakes and ponds differ somewhat from each other, and from those of streams and rivers, still they are only the collected products of these latter, which they consequently greatly resemble.

Ground water, proceeding directly from the earth, offers more distinctive characteristics, shared, generally, by all its subclasses.

As regards the relative salubrity of water drawn from minor streams and that from large rivers, it would seem that they stood nearly upon a par. their principal difference, as already mentioned, being that of size. At first sight it would appear that the smaller streams, situated near the headwaters of the larger ones, or rivers, would possess a higher degree of purity from the fact of the water being collected from a comparatively wilder and more thinly populated district. This is not, however, of necessity, the case, because the river, although passing through a denser population, affords by its greater volume a greater degree

of dilution. The true criterion in respect to this view of the question would seem to be the density of population per square mile of drainage era, together with the proximity of its center of area to the stream itself. The farther the bulk of the population is from the stream or river, the greater the chances of purification by natural filtration. Large rivers are apt to have towns directly on their banks, which drain all their sewage immediately into the stream.

Large rivers, being made up principally of the smaller streams, would appear to form the general average of all their feeders. It must be borne in mind, however, that besides the yield of the smaller streams the river is partly fed by direct surface wash from its immediate banks, thus imparting to it a somewhat modified character, distinct from that of the smaller brooks emptying into it, and causing the water of the large river to be, as a general rule, softer and warmer than that of its small feeders. Generally speaking, however, in the large river you get *all* the water and *all* the impurities, thus making, as already stated, a pretty fair general average of the whole, while of the smaller feeders some will have a greater and some a less degree of concentration of impurities than the average.

There is another point worthy of note as regards the relative quality of the water taken near the mouth of a great river, or from the smaller streams near its source. All impurities entering such streams or rivers have a greater chance of being exterminated by oxidation, by the lower forms of organic life and by fishes, the longer they remain exposed to these agencies. Hence near the mouth of long rivers we have a right to assume that many of the impurities which entered near the headwaters have been destroyed during their long passage toward the mouth. On the other hand, this prolonged sojourn has increased the probability of development of disease germs which have escaped actual destruction. The question then comes up: Had we better take our impurities and disease germs fresh or stale? And the answer

would probably be: Fresh, if we must take them at all, and cannot trust to time for their destruction.

There is another point of difference between the two classes of streams, which, although possessing an engineering rather than a sanitary character, it may not be amiss to refer to here. In the case of the small streams a greater necessity will generally exist for storage, in order to secure a uniform supply, while gravity can be more often counted upon as a motive power than in the case of the large river, where storage is seldom needed, and where pumping is almost invariably necessary; a notable exception being that of Washington, D. C., where the water of the Potomac flows into the city by gravity.

Ground water may be drawn from shallow or deep seated wells—the latter often improperly called *artesian*—from galleries, or directly from springs at the point where they burst forth from the earth.

Shallow wells are supplied by the rain which falls and soaks into the ground in their immediate vicinity. In seasons of much rain the level of saturation is comparatively near the surface; in seasons of drought the level descends as the water gradually drains off to the nearest valley.

While an isolated shallow well may afford water of excellent quality and considerable relative coolness, such wells situated in towns and villages, or even when located near the cesspools of solitary dwellings, constitute what upon the whole must be considered the most objectionable supply in common use. Their hardness and saltness when compared with neighboring springs are a good indication of their relative contamination by human refuse. These qualities are observed to increase with time, and the growth of the village—a striking corroboration of what has been advanced above.

Deep seated wells, and springs, may be fed by rain falling on

far distant points. The water from these wells is apt to be impregnated with earthy salts, and therefore to be hard, frequently to the extent of unfitness for domestic uses. The temperature is apt to be higher than that of shallow wells.

The supply from deep wells is more abundant and steady than from shallow ones, the volume of supply being more dependent upon depth than diameter ; indeed increased diameter only affords greater storage in any given case.

Ground water is frequently obtained from drains or filtering galleries, or lines of pipe with open joints, chiefly located near and parallel to and lower than rivers, which galleries intercept the water flowing through the ground toward the river, and which probably are also fed, to some extent, by the water of the river itself, leaching back to the drain, gallery or pipe line.

Water in considerable quantities is sometimes collected from springs, and conveyed away immediately as it bubbles up from the ground. The new water supply of Havana, Cuba, is a notable instance of this, where some four hundred springs, furnishing over five millions of cubic feet per 24 hours, have been collected about ten miles from the city, and the water conveyed in an aqueduct to a distributing reservoir, whence it is delivered to the city in cast-iron pipes. Such a supply seems likely to be the purest that can be obtained. Nevertheless, spring water is apt to be somewhat hard from the amount of earthy salts frequently held in solution.

It will be seen from the above that the question of the relative purity of different classes of water is a very complicated and uncertain one, not admitting of a general solution, but involving the consideration of a great number of special cases. Ordinarily the choice in any given instance is very limited, most towns having but few sources of supply to select from. The choice is ordinarily further controlled and limited by questions of quantity and cost, so that it seems hardly worth while to consider the

subject under its general aspect at all, but simply to make a special study of each special case.

Quantity of Water.

Next in importance to *quality* comes the question of *quantity*. It will be observed that the growing tendency is to increase the amount allotted per capita per diem. It is found to be necessary to make abundant provision for the future growth of the town to be supplied, to anticipate an increasing individual use of water, and also to provide for the yearly increase of leakage, consequent upon the gradual deterioration of the work and of the house plumbing. This latter is a fruitful though often overlooked cause of a diminishing supply.

In general it may be said that a hundred gallons per twenty-four hours per capita, to be consumed in ten hours, with a liberal allowance for future growth of population, is a safe but not extravagant estimate. We frequently hear of a town finding that its water supply has become inadequate; we never hear of one suffering from too great a one. The control and diminution of waste are now occupying a great deal of attention, particularly in England, where the density of the population renders strict economy necessary. Frequently, no doubt, the best and cheapest way of increasing a deficient water supply would be to reduce waste, by the use of meters and other means for securing the co-operation of consumers.

From the purely engineering point of view the principal interest involved in the hourly supply is its connection with the size of pipes required for its delivery. A hundred gallons per head per twenty-four hours if delivered in ten hours, is at the rate of ten gallons per head per hour, or about 0.00037 cubic foot per second.

It has just been implied that quantity is secondary to quality, but in studying a water supply project the first step is to decide upon the quantity necessary or desirable to obtain. This fact

being settled, the question will naturally follow, How can we ascertain what the yield of a given stream will be? One way is by gaging, and this should be always done, choosing both the driest and wettest seasons for the purpose. But it is evident that all this takes time, and even a year's continuous gaging would not be considered as conclusive in any case where the demand nearly approaches the probable supply, because we must calculate on an occasional year or two of very exceptional drought. Another way is to make a survey of the area which drains into the stream under study, above the point at which it is proposed to take the water. This area, combined with the rainfall, known or assumed, and a general knowledge of the character of the watershed, furnish reliable data for calculating the approximate yield of the stream. Here again, however, we are confronted with the necessity of consuming much time, for although the survey can be rapidly made, the records of rainfall require at least as much time as does gaging. Fortunately in many cases we can make pretty close estimates of the amount of water probably derivable from a given area, by using data already collected for neighboring districts, and at any rate we can always make reasonable assumptions when once we know the number of square miles of territory which drain into our stream—the liability of such assumptions to be correct increasing with the area, for a large area is less subject to special variations from local causes than a small one.

The average yearly rainfall in the Croton basin, which furnishes by far the larger part of the water supply of the city of New York, is about 46 inches. Long experience shows that in this basin each square mile of watershed, or drainage area may be safely counted upon, one season with another, to furnish one million of U. S. gallons per twenty-four hours, or 365 millions per year. On the other hand, a precipitation of 46 inches gives very nearly 800 millions per square mile per year. Hence, in the Croton basin about 46% of the total rainfall is found to be available for water supply.

It must be borne in mind that the above yield represents the *yearly average*, which may easily vary forty times either way for any given shorter period. This fact establishes two important points in regard to water supply. First, the necessity of adequate storage reservoirs to convert this yearly average into a daily one; and secondly, the necessity, in the interest of safety, to give these reservoirs ample overflows, or spillways, in order to provide free escape to the surplus water which may flow into them in immense volumes during freshets.

These considerations bring us naturally to the question of storage, a most important and by no means simple one. The amount of storage necessary to insure a regular daily supply varies of course with the extent of the watershed in proportion to the demand. The larger the area, the smaller may be the storage. In some exceptional cases the supply may be so great that its absolute minimum yield is greater than the maximum demand, and in such cases no storage is necessary. On the other hand cases so unfavorable may possibly occur when the total yearly average is needed, and this leads to a maximum storage capacity.

Let us consider such a case, and suppose a community which requires a supply of 10 million gallons a day from a drainage area of 10 square miles, and follow the course of events through an entire year. The year will be divided into three periods: The period of average flow, the period of drought, and the period of over-supply.

The period of average flow will be that in which the daily yield of the stream is exactly equal to the daily draft—in our supposititious case 10,000,000 gallons. The period of drought will be that in which the yield is less than the above, and the period of over-supply, that in which it exceeds it. These last two periods will, of course, vary in intensity, through indefinite gradations.

In order that the storage capacity should be ideally perfect, it would be necessary to so proportion it that at the commence-

ment of the period of drought the reservoir should be exactly full and of capacity sufficient to bridge over the interval between the drought and the commencement of the period of average yield. At the commencement of the over-supply or freshet period, the reservoir should be completely empty, and of capacity sufficient to receive and retain all the surplus water until the period of average flow was again reached. Not a drop should ordinarily escape except through the supply pipes, and an overflow or spillway should be unnecessary except to provide for extraordinary contingencies, such as cloudbursts, etc.

It is clear that such an ideal state of things is impossible of realization. It would be based upon a regularity of regimen that could never occur except, perhaps, by chance, during a single year. Even in periods of extreme drought (and this extreme is a variable) there would be some water flowing in the stream, and the storage reserve would need to be drawn upon only for the difference between this amount and the daily supply; while during freshets the amount to be stored would be reduced by the daily supply being drawn off. Moreover, besides the average intensity of droughts and freshets, there come cycles of still greater intensity, all of which circumstances are controlling factors in the problem.

In the case assumed the only way to secure the total flow, so that none shall pass to waste over the spillway except in the case of a cloudburst or of some other phenomenon, and at the same time to provide for droughts of maximum intensity, is to construct a reservoir or reservoirs of capacity to contain the total yearly yield of the stream, and to commence the use of the supply with a full reservoir, so that there shall always be a year's supply ahead.

This treatment of the problem is certainly a heroic one, and has probably never been fully carried out in practice, although the city of New York is reaching well on toward it, in the vast

storage works executed and contemplated in the Croton basin. Fortunately so unfavorable a case as the one assumed, when the total yield of the stream is needed, rarely presents itself, and in the majority of cases there is an excess, more or less considerable, of the supply over the demand.

The solution of the problem of storage capacity lies between the two extremes above instanced, in one of which no storage is necessary, and, in the other, when it is necessary to have capacity for the yield of the whole year.

It is evident that we cannot say, *à priori*, of any proposed water supply, that storage for so many days will be necessary, or sufficient, without knowing at least approximately the total yield as well as the desired consumption. In cases where close calculation is needed, as when it appears necessary to utilize the greater part of the supposed supply, the proper course to pursue is to ascertain, by actual survey, the drainage area; to ascertain, by rational assumption when direct observations are lacking, the average precipitation, and then allow from one-quarter to one-third of the same as available, backing these data by gagings, as complete as may be possible, of the stream, and calculate the storage capacity accordingly.

I have confined myself in the above to a general view of the principles involved in planning storage reservoirs, nor do I think it wise to enter into more elaborate calculations, as they might lead the inexperienced to suppose that the problem really admitted of a general mathematical solution. Such is not the case, and unless the known factors point clearly to self-evident assumptions, great caution and much study should be bestowed upon the fixing of the data on which the design of an economical and satisfactory water supply is to be based. One thing is certain: Except for economical reasons there is no danger of having too great storage capacity. I do not happen to recall an instance of a community suffering from the possession of too much stored water,

while the want of enough of it is proving a serious trouble to cities and towns all over the country.

I have already adverted to the matter of adequate spillways for discharging the floodwaters of freshets. There is no uniformity of practice for the dimensions of these all-important adjuncts, and it is probable that the great majority of those now in existence have been proportioned by guesswork, or, "upon general principles." This is all wrong; and in this, as in all other questions of design, we should first ascertain what conditions our structure will have to fulfill, and then dimension it accordingly.

The capacity or open area of a spillway, is made up of its length and height of notch. It must be large enough to pass all the water of extreme floods without danger of over-topping the dam. Forty times the average flow, or 40 million of gallons per square mile and per 24 hours—or 62 cubic feet per square mile per second—is none too liberal an allowance, particularly for earthen dams. For this amount of water we have the two simple approximate formulæ to determine the length and depth of notch of a spillway, the depth being counted from the level of the lip of the dam to the surface of still water in the reservoir:

$$L = 20\sqrt{A}, \tag{1}$$

$$D = \sqrt[3]{A} + C \tag{2}$$

in which $L =$ length in feet, $D =$ depth in feet, $A =$ area of watershed in square miles, and $C =$ a certain additional height above the water in the reservoir, depending upon the character and construction of the dam. If we should wish to provide for a different amount of water, we must generalize formula (2), writing:

$$D = \frac{\sqrt[3]{Q^2}}{16} \times \sqrt[3]{A} + C, \tag{3}$$

in which $Q =$ cubic feet per second per square mile.

Dams.

Large reservoirs are generally formed by building a dam across the valley of the stream furnishing the supply. Naturally,

the narrowest point is chosen, but further investigation may prove such point to be not the most favorable one. A solid foundation is the first requisite, and sometimes firm rock is found so much nearer the surface at a point where the valley is wider that a dam built there would be actually shorter than at the narrower point, besides saving the extra excavation to get down to solid bottom. It is abundantly worth while to devote considerable time in exploring and surveying, before fixing definitely upon the location of the proposed dam.

In examining the character of the foundations, I think that test pits furnish the only trustworthy information. At great depths, these would be very expensive, and recourse is generally had to drilling. This furnishes good indications when properly interpreted, but also has occasioned many expensive misconceptions of the ground. The test pit remains the only sure means of ascertaining what is below the surface.

The character of the ground will determine the class of dam which should be built. If good rock bottom is to be found, a masonry dam will be the best, and perhaps not much more expensive than a properly constructed earthen one. All the elements of a masonry dam are more fixed and precise than those of an earthen dam can be, so there is less necessity for piling up what may in reality be redundant work, to provide for contingencies which we cannot exactly determine quantitively.

Masonry dams may be divided into three classes ; low, medium and high. Although the lines of demarcation are somewhat vague, we may class all dams less than thirty feet high as low, those between thirty and sixty as medium, and all those above sixty as high. Before commencing our investigations it will be extremely useful to establish certain data.

Calculation shows that the equation of equilibrium of a dam with vertical faces is :

$$Wx^2 = 20.83\ H^3, \qquad (4)$$

in which $W =$ weight in pounds of a cubic foot of the masonry, $H =$ the height of wall, and $x =$ its thickness, both in feet. The weight of a cubic foot of water is assumed at 62.50 pounds. From this equation we derive :

$$x = \frac{4.565\ H}{\sqrt{W}}. \qquad (5)$$

These equations show that the overturning moment varies as the square of the height, and the resisting moment as the square of the thickness, and the square root of the density of the masonry; while the value of x, the thickness, varies as the height, and inversely as the square root of the density, of the wall ; that is to say, from (5) we deduce :

$$\frac{x\ \sqrt{W}}{H} = 4.565,$$

a constant.

If we assign 125 pounds as the unit weight, or weight per cubic foot, of the masonry, we find :

$$x = 0.41\ H. \qquad (6)$$

This is the value of x for exact static equilibrium. We may obtain whatever factor of safety we wish by simply multiplying the square of 0.41 by such factor and extracting the square root of the product. Thus, suppose we wish a factor of 2.5. Operating as above, we find ;

$$x = 0.648\ H. \qquad (7)$$

As the assumption of weight is somewhat arbitrary, we may for simplicity write (7) thus :

$$x = \frac{2\ H}{3}; \qquad (8)$$

that is to say, a "plumb" wall to resist water pressure should be twice as thick as one to resist average earth pressure.

However, dams are not built plumb. They generally have vertical backs, toward the water, and battering faces. The readiest way to transform a vertical or plumb wall into a trapezoidal one of equivalent resisting moment is to follow Vauban's

principle, that all equivalent walls with vertical backs have the same thickness at one-ninth of their height from the bottom. This rule holds very closely good within wide limits.

As an application, let us take the case of a wall to sustain water, 27 feet high. If vertical, its thickness should be 18 feet for a factor of safety of 2.50, and we would have the rectangle *A B C D*, shown in Fig. 16. Transforming, according to Vau-

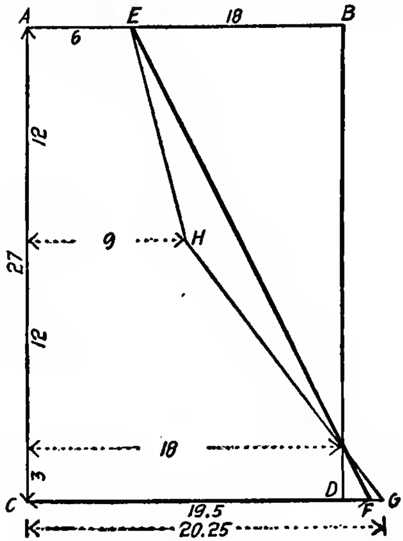

FIG. 16.

ban's rule, to a trapezoidal section, with top width of 6 feet, we get the figure *A E F C*, of which the bottom width *C F* is 19.50 feet. Verifying the comparative stability of the two sections, we find that of the trapezoidal one to be within about 1½% of the rectangle, while its area is nearly 30% smaller.

Such a section as *A E F C* is obviously awkward, presenting a top-heavy appearance, from the redundant thickness of its upper half. Some such section as *A E H G C* would be pref-

erable; it has still less area and not much less stability than the trapezoid $A\ E\ F\ C$.

Passing to dams of medium height, let us take for an example one 54 feet high (Fig. 17). The proper thickness, if vertical, per

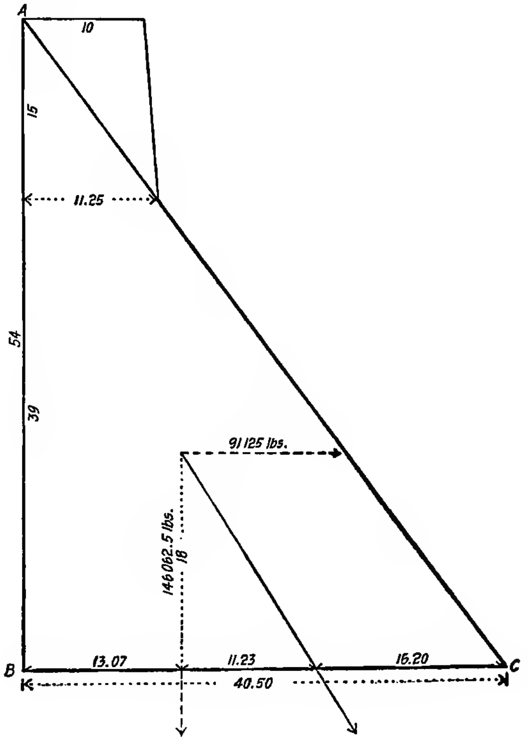

FIG. 17.

formula (8) is 36 feet. Transforming the rectangular section into a *triangular* one, by Vauban's rule, we obtain the right-angled triangle $A\ B\ C$ which possesses certain interesting properties. In the first place, the triangular section has only

about 85% of the stability of the rectangle. Secondly, its base will always be three-quarters of its height. Thirdly, the resultant of its weight combined with the thrust of the water (still assuming the specific gravity of the masonry to be 2) will always cut the base about 11% within its "middle third"; while, of course, the action of the weight alone will cut the base exactly at the inner extremity of the middle third.

Evidently such a section is impossible in practice, because it involves a top width of zero. Let us give it a top width of 10 feet, with a face batter of an inch to the foot to the upper part. This batter will always intersect the hypotenuse of the triangle at a distance from the top equal to one and a half times the top width, whatever that may be.

This composite structure has a resisting moment about 8% less than that of the rectangular one of the same height, and base of 36 feet. It still has a factor of safety of 2.42, with the above relative densities of masonry and water, while the section is about 40% less than the rectangular one. Fig. 17 shows all dimensions, and the triangle of forces. It will be noted how the addition of the upper trapezoid modifies the points of application of the pressures.

In regard to all dams, high or low, we may lay down the leading principle that the line of pressure should always pass within the middle third of the base, especially the line which corresponds to a full reservoir; that is, the line which is the resultant between the weight of the dam itself and the thrust of the water. In very high dams it is not sufficient that this condition be fulfilled for the base only: it must hold good also for any horizontal bed between the base and the top, because in such dams, in order to economize material, the face is given the form of a convex curve, and if this convexity be too great it will occur that, while the base may have a satisfactory width, some of the upper beds parallel to it will not. The object in designing a high dam is to give the section such a form that it shall be a "section

of equal resistance," because this is always the section of greatest economy.

The problem is further complicated in the case of very high dams by the fact that the resistance to overturning is not the only thing to be considered. We must also determine whether the area of the lower beds is sufficient to resist the crushing strain brought upon them by the weight of the superincumbent mass. In making this investigation it is obvious that the first step will be to fix a proper limiting unit strain, or admissible pressure, per square foot upon the masonry. This limit depends upon the nature of the material used and also upon the views of the designer. For ordinarily good masonry, 15,000 pounds per square foot would be considered a conservative limit, being a trifle over 100 pounds per square inch. If the resultant of the pressures cut any bed exactly in the middle, we could ascertain the pressure per square foot upon such bed by simply dividing the whole weight of the mass resting upon it by its length. But when the resultant moves from the center, the strain is no longer evenly distributed over the entire bed, but is intensified upon that portion comprised between the point where the resultant cuts the bed and the nearer extremity of the same, reaching its maximum intensity at the extremity itself, or as we should say, at the *nearer toe.* The investigation of this varying strain, which increases in proportion as the resultant approaches the nearer toe, is somewhat obscure, and rests upon assumptions of somewhat unsatisfactory demonstration. The following two formulas may, however, be accepted as reliable approximations to the truth, within the limits occurring in ordinary practice :

$$P = \frac{2\,W}{3\,D}, \tag{9}$$

$$P = \frac{4\,W}{L^2}\,(L - 1.5\,D), \tag{10}$$

in which :

P = pressure, in pounds, per square foot.

L = length of given bed, in feet.

W = weight, in pounds, of mass above given bed.

D = distance, in feet, from point of intersection of resultant with given bed, to nearer extremity of same.

Formula (9) is used when D is equal to or less than $\frac{L}{3}$. Formula (10) is used when D is equal to or greater than $\frac{L}{3}$. When $D = \frac{L}{3}$, either formula gives unit strains equal to twice the total weight above bed, divided by its length.

We have then the three following conditions which the proper section of a high masonry dam should fulfill: *First,* the lines of pressure should lie within the middle third of all beds. *Secondly,* the maximum unit strains should not exceed a moderate fixed limit. *Thirdly,* the section should be one of equal or nearly equal resistance.

Now then, in the light of what has been already established, let us feel our way toward the proper design for a dam 160 feet high fulfilling the above three conditions. Let us assume a density of masonry double that of water, a limiting unit strain of 15,000 pounds, and, as is usual in such cases, let us consider a length of one foot of dam, so that the area of our section in square feet will represent an equal volume in cubic feet.

Knowing that one of the necessary conditions is that the resultants shall lie within the middle third of all the horizontal beds which we may suppose to divide the section, we feel sure that we cannot go far wrong in first laying down the right-angled triangle $A\ B\ C$ (Fig. 18), of base equal to three-quarters of the height, or 120 feet for the total height of 160 feet. Desiring a top width of say 20 feet, we lay off the same from A, giving the face of this portion of the section a batter of $\frac{1}{12}$. This batter we already know will strike the hypotenuse 30 feet vertical from the top. Now as we know that the effect of placing this top story

upon our triangle will be to draw the line of vertical downward pressure, due to weight of masonry alone, away from the center of gravity of the triangle *A B C,* and therefore outside of the "middle thirds" on the water side; and, also, anticipating a

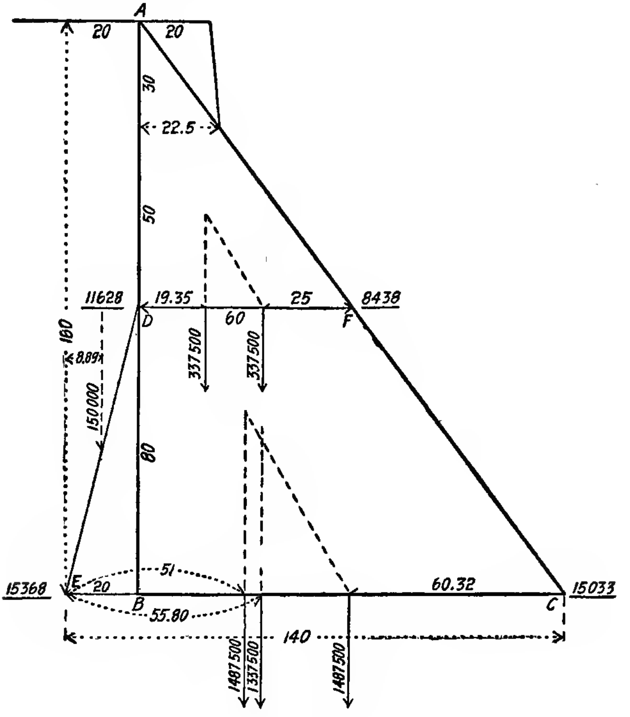

FIG. 18.

little the knowledge which we shall presently acquire, we give the back of the section, from a point 80 feet from the top, an outward flare of one to four, which is shown in the figure by the small triangle *D E B,* and which increases the total width of base to 140 feet. The section is thus divided into three trapezoids,

respectively 30, 50 and 80 feet high, with corresponding widths of 20, 22.5, 60 and 140 feet.

Next we determine, either graphically or by simple calculation, based upon the properties of similar triangles, the points where the line passing through the center of gravity of the superincumbent mass cuts the imaginary bed *D F*, and also where it cuts the same when shoved forward by the thrust of the water, acting at right angles to it. These points are shown in the figure to be respectively at 19.35 ft. and 25 ft. from *D* and *F*. The figure shows also the intensity of the strain in pounds at these points, obtained by multiplying the total area above *D F* by 125, the assumed weight in pounds per cubic foot of masonry, although the calculations were actually made in units of volume, for the sake of rapidity and ease, counting the weight of a cubic foot of masonry as 1, and that of water as ½.

We next proceed in the same way in regard to *E C*. Here we have first the point where the line passing through the center of gravity of the entire section *A E C* cuts the base *E C*, 55.80 ft. from *E*, and which corresponds to an empty reservoir; and secondly, the point, 60.32 ft. from *C*, where the resultant of the weight of the section *A E C*, *plus* the weight of water resting upon the inclined surface *D E*, combined with the forward thrust of the water acting under a head of 160 ft., cuts the same base *E C*, which point corresponds to a full reservoir. The intensities in pounds of all these strains are shown on the figure.

Our design now fulfills one of the imposed conditions. The lines of pressure lie well within the middle third, except at *D*, where the condition is not so binding. It remains to see how it complies with the second one. For this, we recur to our formulæ (9) and (10); and first to ascertain the unit strain at *D*. For this we employ (9), within which the case just falls. Substituting numerical values, we have:

$$P = \frac{2 \times 337500}{3 \times 19.35} = 11628 \text{ lbs. per sq. ft.}$$

For the strain at F we nse (10):

$$P = \frac{4 \times 337500}{3600}(60 - 37.5) = 8438 \text{ lbs. per sq. ft.}$$

Passing to $E\ C$, we have for maximum strain at E, when reservoir is empty,

$$P = \frac{4 \times 1337500}{19600}(140 - 83.70) = 15368 \text{ lbs. per sq. ft.}$$

For maximum strain at C, when reservoir is full :

$$P = \frac{4 \times 1487500}{19600}(140 - 90.48) = 15033 \text{ lbs. per sq. ft.}$$

The maximum strains are given on the plan by the underlined figures at D, F, E and C.

Examining our design, we see that although it practically satisfies the first two conditions demanded, it is by no means a section of equal resistance, for the strains at D and F are far less than those at E and C. Evidently the upper bed $D\ F$ is too wide.

As a further step in our tentative process, I will now offer, Fig. 19, a section suggested by Señor D. E. Boix, in his excellent treatise on "*La estabilidad de las construcciones de Mamposteria*," as a general approximate type for high masonry dams. Beginning at the top, the skeleton of this design consists of a right-angled triangle $A\ B\ C$ of base equal to two-thirds of its height, which height Señor Boix makes a constant of 24 meters, or say 80 ft. From B the back slopes to D with a batter of $\frac{1}{3}$, and from C to E with one of $\frac{3}{4}$. The skeleton of the design, therefore, in this particular instance of a total height of 160 ft., consists of a trapezoid $B\ C\ D\ E$ 80 ft. high, 140 ft. wide at bottom and 53.33 at top, surmounted by a right-angled triangle 80 ft. high. This upper triangular part is a constant, for all sections; the variation occurring in the lower trapezoidal part, according to the total height of dam. As a practical detail, the upper part is surmounted with another triangle, giving the section a proper top width. I have assumed a top width of 20 ft., with batter $\frac{1}{12}$, to correspond with previous example. These di-

mensions, with the triangles of forces and strains per square foot at the points *B, C, D* and *E*, are shown in the figure.

This may be considered an improvement upon the previous design. Its area is about 5% less and there is a much better distribution of strains. Its resisting moment is a little less, being as 2.87 to

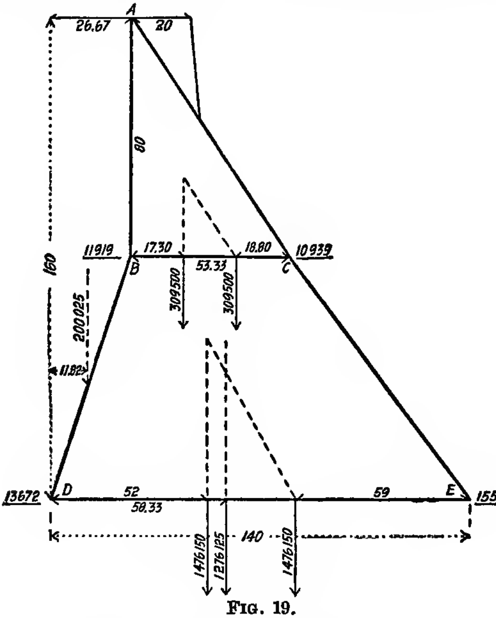

FIG. 19.

3.10, but the coefficient of stability is sufficient. Its practical superiority lies in the 5% of economy. In a careful final study of a high dam, it would be necessary to pass a greater number of horizontal beds through the section, and calculate the unit strains at each extremity of each. The result would probably lead to a more pronounced curve on the face, with a corresponding saving of material.*

* It is well, however, that the upper part of the dam should have a greater proportional strength than the base, because it is exposed to a greater degree of wave action.

An example of a well-proportioned dam, 80 feet high, is given, with all its dimensions and the triangles of forces, in Fig. 20. Calculations, similar to those used already, show maximum strains to be as follows: At E, empty reservoir, 8,677 pounds per square

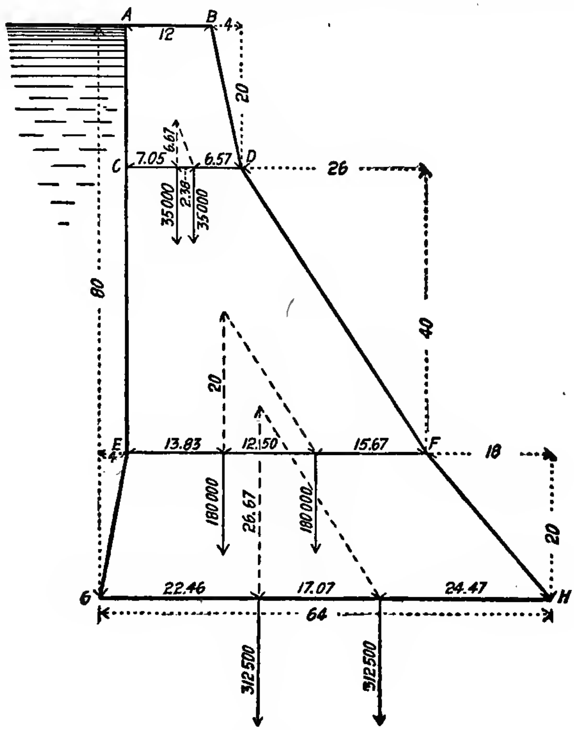

FIG. 20.

foot; at F, full reservoir, 7,607. At G, empty reservoir, 9,250 pounds per square foot; at H, full reservoir, 8,334. The water pressure on E G has not been counted.

In designing a high dam, it should be borne in mind that too much confidence is not to be placed upon the formulæ

used in calculating the unequally distributed strains, and that the further their resultant moves from the center of the beds the less they are to be depended upon. As a further complication Señor Boix, in the work already mentioned, calls attention to the fact that, instead of considering only the vertical component of the resultant acting upon a horizontal bed, we should consider the resultant itself acting upon an imaginary bed inclined at right angles to it. He shows that this gives a more or less augmented unit strain. In an example which he gives of a dam about 100 feet high, the pressures thus calculated exceed those calculated upon the hypothesis of vertical action upon a horizontal bed by 10% to 17%. It would greatly, and I think unnecessarily, complicate the problem to treat it in this manner, for the weight of the superincumbent mass of masonry and the angle of the resultant are interdependent, and tedious processes by trial and error would be needed for each bed. The circumstance is merely referred to in order to show the necessity of keeping well within the margin of safety, for it must be borne in mind that a dam once built cannot readily be remodeled, and should stand intact and without material repairs as long as the town does which depends upon it for its water supply. The prototype of the modern high masonry dam is that across the valley of the Furens, near St. Etienne, in France. The perfect success of this dam is no proof that its section is suited for one of indefinite length, for it is itself very short, and wedged in between the rocky sides of the narrow valley which it spans, thereby receiving great additional strength from these lateral supports.

As regards the plan of a high masonry dam—that is, whether it should be straight or curved, with the convexity up stream—it cannot be said that a curved plan is necessary, nor, on the other hand, can it be denied that such plan is an element of strength, particularly if the dam be short. By adopting this form, in case of a slight movement occurring when the dam comes to its bearings under pressure, the character of the strain will be always

compressive; while, if the axis be straight, any forward movement, however slight, will produce tensile strains, which are always to be avoided in unelastic materials like masonry.

Concerning the further details of the design of high masonry dams, they should stand upon a base or pedestal of which the top is level, more or less, with the surface of the ground, with ample offsets or projections beyond the toes of the dam proper, particularly on the downstream side. The excavation should go through the superincumbent earth, to and into the solid rock. The sides of the base should be vertical and, in the rock, built or packed close against the sides of the excavation. In earth, the space between the vertical sides of the base and those of the excavation should be filled up solid with closely compacted and puddled material, so as to leave no vacancies. The angle which the outer slope of the dam makes with the horizon should never be less than 45°, so as to avoid sharp and weak edges at the points of maximum strain. Whenever sufficient material can be obtained from the excavation or from borrow pits, a sloping bank of well-compacted earth should be placed against the back of the dam for at least a quarter or a third of its height from the ground, and protected by riprapping. This bank has a double object: it not only impedes leakage, but by establishing a permanent thrust, or at least support, against the back of the dam when the reservoir is empty, it diminishes the range of pressures existing between a full and an empty reservoir. All of these features are shown in Fig. 21.

Before leaving this question of design, there is a recommendation which I think it well to make, not only in regard to high masonry dams, but to all other engineering structures as well. It is this: After carefully proportioning the work according to approved methods of calculation, *take a good look at the drawing*, and see if it LOOKS right; if not, there is probably something wrong in the calculations, and they had better be gone over to see where the mistake is.

Where rock foundation is not obtainable, the best kind of dam will be an earthen one, with masonry core or center wall. No earthen dam can be considered safe that is not provided with such wall, carried down to a water-tight or comparatively water-tight stratum. The masonry center wall is the only sure and permanent means of cutting off percolations through the bank, and for this reason it should be carried well down, on the principle that the worse the character of the ground, the deeper should

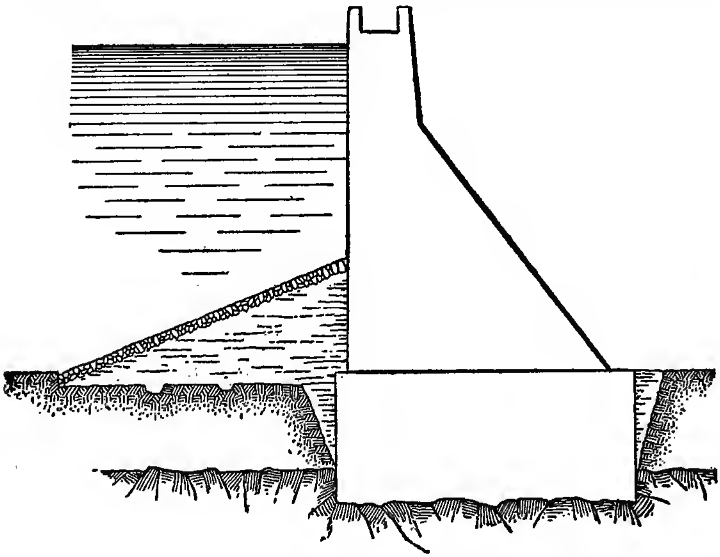

FIG. 21.

be the foundations. It should also be deeply imbedded in the sides of the valley. The worst kind of ground is a loose drift formation, containing many large cobblestones. In such material the wall must be sent "'way down," till perhaps there is as much under as above ground, and the embankment, particularly on the water side, carried far back with a very easy slope, sometimes as much as five or six to one. Clay, sand, and fine, compact

gravel are the best bottoms to build on; quicksand is also excellent, when lateral escape can be prevented, as it usually can be except when very near the surface.

Not only does the masonry center wall prevent percolation, but it also affords the means of making perfectly secure connections with all the accessories of the reservoir, such as culverts, piping, gate towers, etc. The failure of many earthen dams has been due to water following along outside the culverts or pipes by which water was drawn from the reservoir, a circumstance which cannot occur when these appliances are bonded in with a tight or even partially tight masonry wall, extending from flank to flank of the valley, and carried down to a good bottom.

Although this wall is supported on both sides by the embankment, it should be of sufficient thickness to withstand any unbalanced pressures to which it may be subjected. Where highest it may have a bottom thickness of about one-quarter its height, and be drawn in at the rate of about an inch to the foot, but preferably by offsets rather than batter. This would always give a top width equal to one-twelfth the height, which might sometimes lead to unsuitable dimensions, requiring modification. The top should be carried up at least as high as the level of highest water in the reservoir.

The spillway should be proportioned according to the principles already laid down, and if possible some natural depression should be found back of the dam, by which the overflow may be passed over to another valley, or into the same one, lower down. Failing this condition, a massive masonry spillway and apron must be built, generally in the axis of the stream, either with curved face, like the Croton or Bronx River dams, or stepped. like those of the Scranton Gas & Water Co., at Scranton, Pa. I am inclined to prefer this latter mode on its engineering merits, and apart from the fact of its greater cheapness. The section of the spillway will be proportioned according to the principles already laid down for high dams, but it should be even more

massive, for it has to stand the impact of falling water. Its width at its foot should be equal to its height.

In regard to the embankment, it is ordinarily specified that this should be made of selected material. This is certainly recommendable, and the best material the site affords should be used in preference, but it is almost impossible to compel contractors to *sort* the material, and moreover really good stuff is not always within reach, in which case the volume put in the embankment must be increased, to compensate in a measure for its lack of quality. The surface of the bank, on the water side, should be well riprapped, and the lower slope sodded or seeded to grass. The bank should be kept wet by sprinkling while being put in, and should be brought up in horizontal lifts, and not made from a dump, like a railroad embankment. It should be compacted as it goes in ; ordinarily the travel of the carts, wagons and scrapers used will be sufficient for this purpose in connection with thorough sprinkling. The area covered by the embankment, particularly on the water side, should be carefully "floated off," grubbed and plowed, so that the material of the embankment shall come in contact with clean earth. It is also well to dig cross ditches, parallel to the center wall, to secure a still further bonding of the surface of the ground with the embankment. These ditches must be carefully filled and tamped with the material used in the embankment. An excellent form for the inside or water side of an embankment is a series of slopes and berms, forming one or more terraces. Working from berm to berm gives a good opportunity to carry the work up level.

The best way to draw the water from the reservoir is by means of cast iron piping running through the center wall and terminating on the water side in a tower, built in with the center wall, and containing grooves for the reception of stop plank. The pipes should lead on the land side into an easily accessible gatehouse, also built in with the center wall, and each line should be provided with two good gates or valves, the inner one

to be kept always open or partly open, and the outer one used for current operations. Then, should the latter get out of order, the inner gate can be closed, and the necessary examination and repairs effected. It is a good plan to build iron eye-beams into the walls of the gatehouse directly over the gates, so that in case of wishing to remove them the necessary overhead purchase can readily be applied. These ideas are not hypothetical, but have been satisfactorily carried out in the system of storage reservoirs built for the Scranton Gas & Water Company, already referred to.

It will now be well to make some remarks respecting the various classes of work embraced in the construction of dams and reservoirs. In such structures a good deal of concrete is generally employed, especially in foundations and subfoundations, for which purpose it is the best material that can be used. Great care, however, must be taken in its preparation and placing, for there is perhaps a greater difference between good and bad in concrete than in any other kind of masonry. Various proportions have been recommended, and we may lay down as a general principle, that the smaller the stones and the larger the percentage of cement used, the more water-tight will be the resulting concrete. I have obtained excellent results using fine ground Portland cement and thorough mixing, with one of cement, three of clean sand, and six of broken stone. In important work I should not care to use a less rich mixture than this, which, as I may add in passing, was that employed by the late General Gilmore in the rebuilding of Forts Sumter and Moultrie, in Charleston Harbor, with the exception of substituting Rosendale for Portland cement. A sufficient quantity of water should be added to bring the mass to a very decided degree of moisture, while not drowning it with an excess.

The sand and cement should be thoroughly mixed dry; then the stones, previously wetted, are added and the whole mass thoroughly mixed, water being added from time to time till it is brought to the proper consistency. Always keep the bed thin

and avoid getting the material into heaps, which prevents proper mixing. If a mixing machine is used, the sand and cement should still be mixed dry by hand before putting into the machine. If the whole process is done by hand, there should be turned out about two cubic yards of concrete in the work per day of ten hours, per man all told, including all employed in mixing, placing and tamping.

This is about as much yardage as an ordinary gang of stone masons and helpers will do, laying up first class hydraulic rubble, and one of the principal reasons for concrete being cheaper than rubble is the fact of its being done by a cheaper class of labor. Indeed contractors generally bid entirely too low on hand made concrete, as compared with rubble, with the result that they endeavor afterward to do as little of it as possible, trying to get rubble substituted for it. Tamping should be carried on until the mass assumes a jelly like consistency, or at least till moisture is brought to the surface. This is a *sine qua non,* which inspectors much insist on. Prolonged tamping will frequently bring up the water from a piece of concrete which appeared to be hopelessly dry when put in. Bear in mind, that although too much water is bad, too little is very much worse. As a general rule the greater the percentage of broken stone, the more thorough must be the mixing and tamping. If the stone be hard and sharp the concrete will gain by giving it all that it will carry. Concrete may very easily be made too rich in mortar by carrying out the erroneous idea that the smaller the percentage of stone the better. After concrete has been placed, let it remain undisturbed and be kept moist by constant sprinkling for as long a time as possible.

The stone masonry used in the construction of hydraulic work is special in its character, and requires therefor special care in its execution. This fact should be fully explained to prospective bidders before they send in their proposals, or there will surely be remonstrances when the work begins. One principal source of contention is in regard to the size of the stones used. The

work goes on so very much quicker by the use of the largest stones the derrick can lift, that both engineers and contractors are tempted to get in as many as possible. I think, however, that the fact remains the same that a more water-tight wall can be built with small stones. "Small stones" must be taken in a relative, and not an absolute, sense in this connection ; near the foot of a thick wall larger ones can safely be used than in the thinner upper portions; and in hauling and distributing stone on the ground, contractors should be warned to keep the smaller ones handy for the top.

Great care must be taken in bedding and jointing the stones. They should be laid on the natural bed, the best and flattest bed down, and should be made to *swim* on their beds; *i. e.*, they should be susceptible of being swayed from side to side with a bar after they are laid and before being spalled up. No stone should rock when stepped on. When work first begins, stones, after being bedded and pronounced all right by the foreman, should frequently be raised again, so that he may see that, after all, there are many blank patches on the bed of the stone where it did not come in contact with the mortar. All stones, particularly small ones, should be hammered down on their beds. One advantage of using large stones is that they bed themselves to a great extent by their own weight. All the joints should be carefully made up, the invariable rule being, throughout the wall, that there shall be no vacant spaces, but that all that is not stone must be mortar. To this end, stone should not be laid too close together, but ample space afforded for getting the trowel all around them so as to cut the mortar well under and into the beds and joints. When the joints are narrow, they should be filled with mortar, and spalls driven in, as many as the joint will hold, without crowding, the only limit being that in no case shall there be stone to stone. Also be very careful that after a stone has been well bedded it shall not be lifted from its bed by wedging in too many big spalls under it. One of the tests of a good

mason is the way he makes up his joints, fitting in all the spalls he can. Ordinarily, the mason finds it less trouble to fill up the joints with loosely thrown in mortar. This is against the interests of both contractor and company. Needless to say that the work must be well bonded, breaking beds as well as joints. Particular attention must be paid to the mortar, seeing that the sand and cement be thoroughly mixed dry, till it presents a uniform color, without streaks of sand and cement. Good average proportions are 1 cement and 2 sand for Rosendale, and 1 cement and 3 sand for Portland. The greater the proportion of sand the more thorough must be the mixing. Guard against having the mortar too wet, but let it always be worked up with the trowel to a soft pudding in the work. Masons frequently call out that the mortar is too dry, when what it wants is simply more tempering to bring out the moisture. The mason should be constantly tempering up his mortar, and when he has large quantities on hand he must call on his helper to do the same with his shovel. The more you work up and turn over fresh mortar—particularly when Portland is used—the better the results. As to quantity of wall laid up, per derrick, I think that each double drum steam derrick of proper sweep, well tended and constantly fed with materials, should be good for from 30 to 60 or even more cubic yards per day of ten hours. These limits are rather elastic, but perhaps it would not be fair to draw them closer, so much depending upon the size and character of the stones used. I have frequently timed a derrick, to see the number of trips made in a given time, but should be loth to fix a standard. If we allow five minutes to a trip, we should have one hundred and twenty per day. These trips convey not only the larger stones to be set by the derrick in the work, but also all the mortar and spalls needed for bedding and jointing. The governing factor, I think, will prove to be the size of the stones set by the derrick, for it will set a large one about as quick as a small one. One thing the contractor should closely watch in his own interest,

and that is, that a derrick should never stand idle. If it is not constantly in motion he should at once find out what is the matter. Probably it is under-manned at one end or the other, and the gangs need to be increased. Strange to say, in regard to this question of rapidity, that the better the work is done the quicker it is done, because done systematically, and every stroke tells. No work lags so much as slovenly work.

One word more regarding large stones: When used, they must always be swung in place with the derrick, and *never* barred nor rolled to their bed over fresh work. This must be insisted on, or the bond of the mortar will be surely destroyed by jarring and displacing the stones last laid.

In dry riprapping, as in concrete, the best results attend the use of stones of various dimensions, so disposed that the percentage of voids shall be as small as possible.

When laying masonry in wet excavations see that the pumping sump is sunk below the bottom course of masonry, so that the pumps shall not draw the mortar out of the work, and placed outside of the area to be built over. This is a very important, though almost always neglected, point. In backfilling an excavation after the masonry has been built in it, see that the earth is well compacted, dead against the wall, so as to leave no vacancy between it and the sides of the excavation. This is also an important and generally neglected matter.

When work of this kind is undertaken it is indispensable that the principal engineer should give his close personal attention to all the details, particularly at the commencement, because both the contractors and the inspectors must be not only *told,* but *shown,* what is required to be done. Later on, when it may be impossible for him to give his undivided attention to the execution of the work, he should make frequent and prolonged visits to it, at irregular times. It is no reflection on the honesty or ability of the contractors and engineering staff to say that this is

indispensable to secure good results; it is merely another way of saying that if the chief neglects his duty he cannot expect that the others will properly perform theirs.

One of the principal engineering difficulties which dam-building presents is what to do with the water while the work is in progress, and particularly while the foundations are being put in. When dealing with a powerful stream the problem assumes formidable proportions. Since it is indispensable that all dams should have a capacious blow-off culvert, at or near the bottom of the reservoir, I think the best way will often be to get this culvert and all its appurtenances in first, so that the stream may be diverted into it, and thus give no further trouble in average stages of the river. Should the season of freshets intervene when the culvert may not be able to handle all the water coming down to it, the work must be so prepared that the flood may sweep over it without damage. Much forethought is necessary in such cases, and more or less risk can hardly be avoided; the important point being to reduce it to a minimum.

Another difficulty occurs when springs are encountered in the foundation pits. These may be sometimes dealt with by providing a temporary escape for them through the masonry, to be closed afterward when the surrounding masonry has thoroughly set. They can also sometimes be overmastered by pumping, but the ingenuity of all hands is frequently taxed to devise means of getting rid of them, and the firmness of the engineer will also often be severely tested in refusing to allow the work to be commenced without adequate preparation, or to permit a defective foundation, through which a current of water is passing, to remain and receive its superstructure while in that condition.

The above comprise some of the most essential points to be observed in dam-building, but it is impossible to cover the whole ground, and the engineer intrusted with such work will find all his experience and resources called into constant requisition.

One governing principle should never be lost sight of in designing such structures. While thoroughly good work should be exacted in the smallest detail, yet the design of the dam should be such as to provide for disaster arising from overlooked defects of workmanship or from unforeseen contingencies, in such a way as to prevent or at least to minimize the consequent destruction. In the case, for instance, of a cloudburst gorging the spillway of an earthen dam, when the absence of the guardian has prevented the relieving blow-off culvert from being opened in time, there is almost always some point at which the breach can be made with the least bad results, and the design should favor its occurrence at this point rather than in less advantageous ones. I may add that the existence of a stout masonry center wall is always a tower of strength, and may frequently prevent a catastrophe should the dam be overtopped. "And even if the worst should "occur, and the center wall be breached, time, that priceless "element in such cases, would be gained, and the catastrophe "greatly modified. For the only difference between the harm-"less emptying of a reservoir, and a Johnstown disaster, is one "of time." *

NOTE.—In addition to what has been said in the above pages, the reader will find useful information in "Jacob's Storage Reservoirs," revised by the present author, and published in "Van Nostrand's Science Series." Also, as to high masonry dams, see two articles in Van Nostrand's *Engineering Magazine*, Vol. 30, 1884; also "Specifications for Dams and Reservoirs, published by THE ENGINEERING NEWS PUBLISHING CO."

THE END.

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* Discussion of Mr. Desmond Fitz Gerald's paper on "Rainfall, Flow of Streams, and Storage," Transactions of the American Society C. E., Vol. XXVII., page 295, which see also for further remarks about masonry, center walls and capacities of spillways.